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DYNAMICS OF CRESCENT WAVE PATTERNS IN A CHANNEL

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Summary Solving the full set of water wave equations, we perform numerical simulations of evolution of class II instabilities. We reproduce the well known steady horse-shoe patterns and the oscillating ones. For small initial steepness, we identify the existence of a recurrence cycle. We further study the feasibility of experimental observation of such patterns and give an explanation for the selection mechanism.

INTRODUCTION

Instabilities of plane Stokes wave have been studied for many decades. For infinite water depth, modulational instability is dominant for small to moderate initial steepness $(ak)_0$ and leads to a recurrence phenomena for small initial wave steepness $(ak)_0$ and to breaking otherwise[1]. While another class of instability characterized by its three-dimensionality (Mc Lean class II) dominates for larger values of $(ak)_0$ [5,6,7]. This instability leads usually to wave breaking through formation of spilling breakers[9]. The presence of surfactants on the free surface of the flow may cancel the modulational instability, however. Making possible the development of class II instabilities even for small $(ak)_0$. Similarly, for shallow water cases, instabilities of a plane Stokes wave may be dominated by class II [6].

We use hence a numerical model to solve the full set of water wave equations and focus our attention onto the evolution of class II instabilities in the absence of modulational instabilities. We give numerical evidence that the class II instabilities may give rise to a recurrence phenomena similar to the Fermi-Pasta-Ulam recurrence, for small $(ak)_0$, and to breaking otherwise. Analysis of the evolution of several class II perturbation allows us a close comparison with experimental data, including the well known steady horse-shoe patterns[9] and the newly discovered oscillating horse-shoe[2].

The match is impressively good and a further study of the last patterns allows us to suggest an explanation for the selection mechanism that may be responsible for trigging this instability in the case of the Collard and Caulliez[2] experiment (this resonant quintet is theoretically never the dominant one over the class II). We give explanation of the selection mechanism and simulate the experiment.

NUMERICAL EXPERIMENTS

We compute the elevation η of non-overturning surface waves in water of infinite depth using a three-dimensional fully nonlinear method [3,4]. Application of potential theory is assumed. The resulting Laplace equation is inverted by use of Green's theorem and Fourier transform, resulting in a fast numerical algorithm that solve the full set of equations. A special attention is put upon identification of breaking.

In the first step, we compute the elevation $\bar{\eta}$ of an exact plane Stokes waves of steepness $(ak)_0$, where $2a$ is the distance from trough to crest. The frequency of the waves is $\omega_0 = \omega(k_0)$. A small perturbation, $\widehat{\eta}$ of the wave surface, taking the form $\widehat{\eta} = \epsilon a \sin[(1+p)k_0 x]\cos(qk_0 y)$, is added to the Stokes wave train. Here, ϵ is a small number, making the amplitude of the initial perturbation field a fraction of the Stokes waves. $(1+p,q)k_0$ denotes the wavevector. A perturbation velocity potential corresponding to the perturbation surface is computed and added to the initial potential field.

RESULTS

The chosen perturbation corresponds to one of the class II unstable modes according to the McLean diagram for the considered wave amplitude. A three dimensional horse-shoe pattern develops in the numerical tank. Depending on the wave amplitude and on the chosen initial perturbation, either the steady horse-shoes or oscillating patterns appear. While the steady horse-shoe pattern exhibits a steady shape, organized in chess-like order, the new one is characterized by aligned crescents oscillating in time (fig. 2). The spectrum of the wave field shows wave frequencies for the steady wave pattern with $n\omega_0$ and $(2n-1)\omega_0/2$, $n = 1, 2, ...,$. The wave frequencies of the new wave pattern are: $n\omega_0$, and $\sim \frac{4}{3}\omega_0$, $\sim \frac{5}{3}\omega_0$,$\sim \frac{7}{3}\omega_0$,$\sim \frac{8}{3}\omega_0$, etc. Those results together with geometrical comparisons agree well with experimental observations of Su[9] and of Collard and Caulliez[2].

When considering several initial perturbations, all corresponding unstable modes develop and coexist together, the importance of each one depending on its initial amplitude, leading to a chaotic behavior and eventually to breaking.

For small values of $(ak)_0$ (typically $(ak)_0 < 0.17$), the patterns are characterized by a periodic evolution from plane stokes waves into crescent shaped patterns. This recurrence phenomena is similar to the Fermi-Pasta-Ulam recurrence for modulational instability (fig. 1 illustrates this recurrence for the steady patterns). For larger value of $(ak)_0$, the evolution of the perturbation leads to wave breaking.

A further analysis of Collard and Caulliez[2] experiments bring us to suggest a scenario for the appearance of the oscillating patterns. The effect of the wavemaker in a confined channel may generate, through parametric resonance, cross-waves[8]. Nonlinear self-interaction yield to the excitation of high harmonics of the cross-waves. Unstable modes of the class II lying on those eigenmodes are then trigged. An important point is the role of high transverse wavenumbers.

We numerically reproduce the development of steady horse-shoe patterns for an initial steepness $(ak)_0 = 0.13$ and of oscillating patterns for $(ak)_0 = 0.17$ in the same channel while taking the same initial perturbations, in perfect agreement with experimental setup and results.

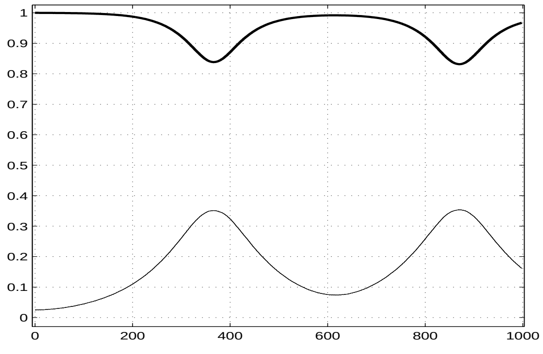

Figure 1. Typical energy evolution for the main modes corresponding to a recurrence cycle ($(ak)_0 = 0.13$), fundamental (thick line) $(k_0, 0)$ and perturbation $(1.5k_0, q)$.

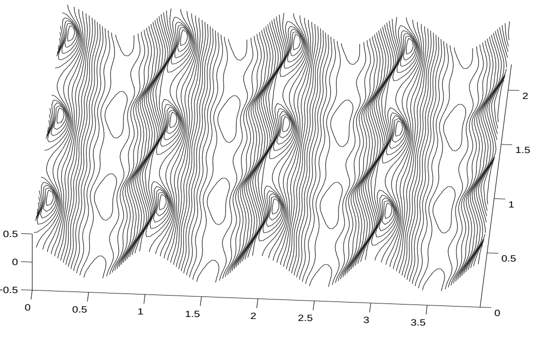

Figure 2. Free surface elevation of oscillating horse-shoe pattern.

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