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NUMERICAL SIMULATIONS OF PARTICLE SUSPENSIONS IN A ROTATING FLOW

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Summary We have developed an algorithm to simulate the motion of suspended solid particles in a rotating cylinder. The hydrodynamic interactions are calculated from the Stokes-flow generated by a point force, with a Green's function that enforces a zero-velocity boundary condition on the surface of the cylinder. We have implemented a parallel version of the algorithm, which also scales linearly with the number of particles. For the time scales of interest, typically of the order of 100-200 rotations of the tube, simulations of a few thousand particles per processor are feasible. The code has been applied to the investigation of pattern formation in a rotating suspension. In these experiments a cylinder filled with a suspension of heavy particles is rotated about a horizontal axis, and a number of different "phases" or particle patterns have been observed and characterized experimentally. We plan to use numerical simulations to elucidate the hydrodynamic mechanisms leading to pattern formation.

Recent experiments [1] have shown unexpected patterns in the spatial distribution when a suspension of non-neutrally buoyant particles is rotated about a horizontal axis. The combination of gravity and rotational lift produce a variety of interesting and surprising steady states. In Fig. 1 we show results for a small number of particles in a short tube. The tube radius is 0.5cm, the fluid viscosity is 0.58 poise and the particle mass density, corrected for buoyancy, is 1.2 g/cc. The parameters are comparable to the experimental setup except that the tube radius is smaller by a factor of 2. At low rotation speeds the particles are confined to a small volume on the lift side of the tube, as shown by the majority of the particles in the upper left-hand picture of Fig. 1. Particles next to the wall are lifted by the rotation of the tube, but in adjacent layers the gravitational forces exceed the viscous drag from the fluid and they slip down to the bottom so that there is a constant circulation of particles. At the slightly higher rotational velocity shown in the first figure, $\Omega = 0.4\mathrm{Hz}$, particles are ejected into the bulk fluid and fall in a more or less semicircular arc rather than straight down, so that the fluid flow in the upper half of the tube must be predominantly clockwise. This flow sets up a strong shear layer across the center of the cell, so that in the lower half of the tube we see a counter-clockwise flow, lifting particles off the base of the tube and returning them to the wall higher up.

At higher rotation speeds these counter-rotating regions grow and become more symmetrical, filling the tube cross section at the same time, as shown in the upper right picture in Fig 1. At still higher speeds the lower counter-rotating region shrinks (lower left) and eventually disappears altogether (lower right), leaving the particles undergoing a more or less rigid-body motion, produced by a delicate balance of rotational, gravitational and hydrodynamic forces. It should be noted that none of these phenomena occur in the absence of hydrodynamic interactions. A single particle simply rotates about an off-center position, while slowly spiralling to the outside wall, where it eventually reaches a limiting trajectory [2].

At the highest rotation rate we have so far studied, $\Omega = 0.9\mathrm{Hz}$, the particles congregate into weak but stable axial bands as shown in Fig. 2. We have previously proposed an explanation for this axial instability based on a differential centrifuging mechanism [2], but the simulations suggest that a complex combination of rotational and gravitational forces is responsible, and that centrifugal forces do not play a major role. It should be emphasized that all the structures shown here have previously been observed experimentally [1], and so far we have only provided a validation of the computational method rather than a discovery of new physical phenomena. We plan to make a detailed sequence of simulations in the coming year, to try to elucidate the mechanisms underlying the pattern formation.

The numerical method is based on the Green's function for a Stokeslet in a cylindrical tube [3]. We have implemented both the Fourier-Bessel representation and the residue sum detailed in Ref. [3]. We obtain identical results when the particles are more than a few diameters apart, but when they are in near contact there are convergence issues with both methods. We assume that since the suspension is dilute, a typical volume fraction is 2%, the short-range interactions are relatively unimportant, but this remains to be verified. We prefer the Fourier-Bessel representation in our simulations since it enables a linear scaling of the computational time with the number of particles.

In conclusion, we have developed a fast numerical method for simulating the dynamics of suspended particles in a rotational flow. The method can treat of the order of 10^5 particles on a moderately sized cluster (32-64 processors). We have reproduced the key qualitative features of the experimental observations at low rotation rates, within the observed parameter ranges. Future work will be aimed at understanding the mechanisms underlying the pattern formation.

References

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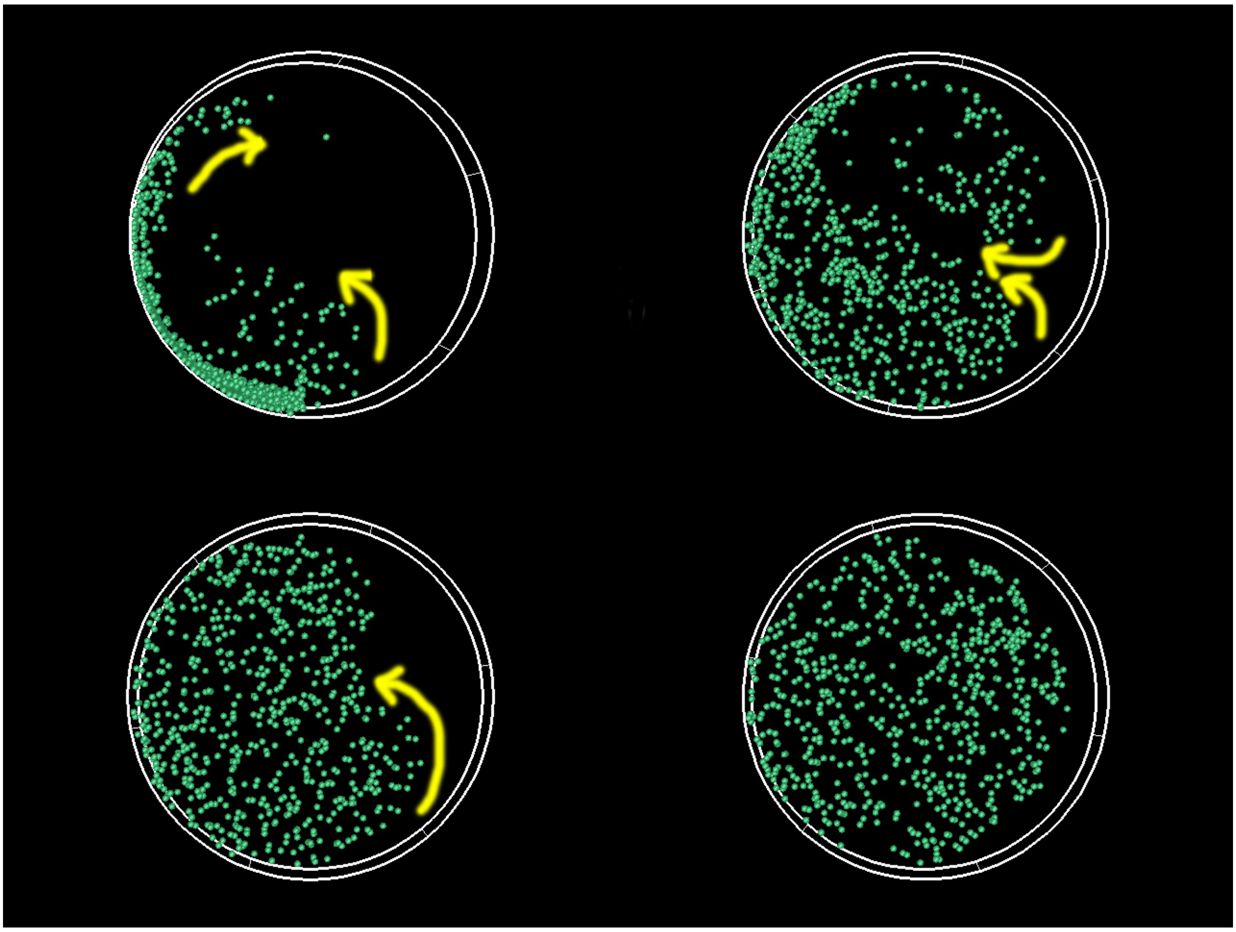

Figure 1. Particle distributions at different rotation rates: $\Omega(\mathrm{Hz}) = 0.4, 0.6, 0.8, 0.9$. At the lowest rotation rate (upper left) a small fraction of the particles are ejected into the bulk fluid by the (clockwise) rotation of the cylinder wall. A counter flow at the base lifts particles in a counter-clockwise flow. At slightly higher rotation rates (upper right) these flows develop into a symmetric pair of semi-circular flows. At higher rates still the lower counter flow becomes small (lower left) and eventually vanishes (lower right) leaving a more or less solid-body rotation of uniformly distributed particles. The yellow arrows are a rough guide to the predominant fluid motions, with respect to the rigid body motion.

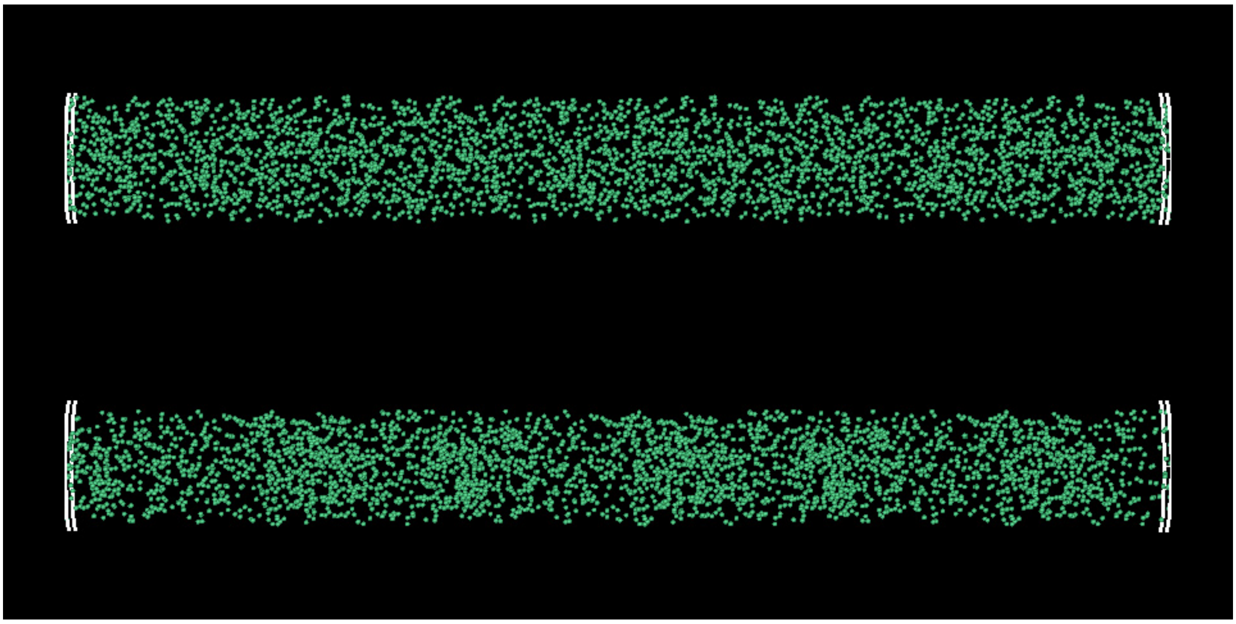

Figure 2. Axial band formation in a rotating suspension. Over a narrow range of rotation rates, $\Omega \approx 1\mathrm{Hz}$, an initially uniform particle distribution (upper) is unstable and permanent axial perturbations develop in the particle concentration. Note that the simulated system been replicated 3 times in the axial direction to allow the eye to pick up the band formation in the lower figure.

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