A second-order slip model for early-transition-regime flows

Small scale gaseous flows have been the subject of significant interest in connection with micro- and nanoscale science and technology. The failure of the Navier-Stokes description as the characteristic system scale (H) approaches the mean free path (ℓ) makes these problems both challenging and interesting from a scientific point of view.

The deviation from Navier-Stokes is quantified by the Knudsen number typically defined as $\mathrm{Kn} = \ell / H$. It is well known that in the regime $0 < \mathrm{Kn} < 0.1$, known as slip flow, the Navier-Stokes description may still be used, albeit supplemented by slip boundary conditions rather than the usual no-slip boundary conditions. In the transition regime $(0.1 < \mathrm{Kn} < 10)$, the slip-flow approximation is no longer accurate, and gaseous hydrodynamics can only be described by more fundamental approaches such as Boltzmann equation solutions or molecular simulations. Due to the considerable complexity and computational cost associated with these alternative approaches, models which extend the applicability of the Navier-Stokes description beyond $\mathrm{Kn} = 0.1$ are highly desirable.

In this paper we present a second-order slip model [1] based on the hard-sphere approximation which extends the applicability of Navier-Stokes description well beyond the slip flow regime. This model is based on rigorous solutions of the Boltzmann equation [2,3], and has no adjustable parameters. We show that the key to obtaining a reliable second-order slip model is accounting for the non-equilibrium layer close to the walls, known as the Knudsen layer. In this paper we discuss how the latter:

1. Impacts the accuracy of the solution
2. Has led previous attempts to incorrect results
3. Is taken into account by the current model.

In particular, we show that the slip model provides a baseline Navier-Stokes solution which does not capture the effects of the Knudsen layer. In fact, as the Knudsen number increases beyond $\mathrm{Kn} = 0.1$, the Knudsen layer whose thickness is of the order of one mean free path, covers an increasingly larger fraction of the physical domain. It is for this reason that previous attempts attempting to fit the velocity profile throughout the physical domain were not successful.

However, despite the increasing importance of the Knudsen layers, the second-order slip model remains accurate and useful well into the transition regime. The effect of the Knudsen layer can be fully accounted for [2] in calculating the average (over the physical domain) flow velocity. Of course the stress field is also accurately captured with no extra adjustable parameters.

We verify our results by comparing with numerical solutions of the Boltzmann equation. In particular, we show that the proposed model is in excellent agreement with direct Monte Carlo solutions of the Boltzmann equation. Three different flows are considered:

steady pressure-driven flow, an oscillatory shear driven flow and an unsteady impulsive start problem. In all cases, excellent agreement is found up to $\mathrm{Kn} = 0.4$. The slip model remains in qualitative agreement with Boltzmann equation solutions beyond this Knudsen number.

References

[1] N.G. Hadjiconstantinou, “Comment on Cercignani’s Second-Order Slip Coefficient,” Physics of Fluids, **15**, 2352, 2003.

[2] C. Cercignani, “Higher order slip according to the linearized Boltzmann equation,” Institute of Engineering Research Report AS-64-19, University of California, Berkeley, 1964.

[3] T. Ohwada, Y. Sone, and K. Aoki, “Numerical analysis of the shear and thermal creep flows of a rarefied gas over a plane wall on the basis of the linearized Boltzmann equation for hard-sphere molecules,” Physics of Fluids A, **1**, 1588, 1989.