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# PROBABILISTIC HOMOGENIZATION OF HYPERELASTIC SOLID FOAMS

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Summary The present study is concerned with a probabilistic homogenization scheme for amorphous hyperelastic solid foams. The scheme is based on the multiple analysis of a small-scale representative volume with a randomized microstructure. The macroscopic stress-strain characteristics are determined by means of a strain energy based homogenization procedure. The results are evaluated stochastically in terms of the stress mean values and the corresponding standard deviations describing the scatter band width.

## INTRODUCTION

Solid foams are important materials in modern lightweight construction. Their main advantage is their low specific weight and their good strength-to-weight ratio (Gibson and Ashby [2]). For reasons of numerical efficiency, components consisting of foamed materials are analyzed in terms of effective properties rather than by a detailed model of the given microstructure. The determination of the effective stress-strain-response can be performed either by experimental or by analytical methods. Although solid foams are amorphous assemblies of pores with different shapes and sizes, most studies in literature use regular, perfectly periodic models for the homogenization analysis. Only few studies of random foams are available which are usually based on large-scale representative volume elements containing a microstructure with a large number of pores obtained by means of a Voronoi technique. It has been shown by Fortes and Ashby [1] that this approach might yield inappropriate results. Instead, direct probabilistic approaches are proposed.

## PROBABILISTIC HOMOGENIZATION

The present study is concerned with a probabilistic approach to the determination of the effective properties of solid foams. The scheme is based on the analysis of a periodic reference microstructure with a prescribed topology. An example together with an appropriate representative volume element (RVE) is presented in Fig. 1. Prior to the analysis, the geometry of the microstructuer is randomized by a probabilistic determination of the spatial positions of the cell wall intersections within prescribed areas of dimension $2\Delta x_i$ (see Fig. 2) using a random number generator. The randomization and subsequent homogenization analysis is performed multiple times in $n$ numerical experiments. The results are evaluated stochastically in terms of the mean stress field and the corresponding standard deviation

$$\bar{\tau}_{ij}^{a} = \frac{1}{n}\sum_{k=1}^{n}\bar{\tau}_{ij}^{(k)} \qquad \text{and} \qquad \bar{\tau}_{ij}^{s} = \frac{1}{n-1}\left(\sum_{k=1}^{n}\left(\bar{\tau}_{ij}^{a} - \bar{\tau}_{ij}^{(k)}\right)^{2}\right)^{1/2} \tag{1}$$

where $\tau_{ij}$ are the components of the second Piola-Kirchhoff stress tensor. Advantage of this method compared to the standard probabilistic approaches based on a single analysis of a large scale representative volume element is the numerical efficiency due to the small mechanical model to be analyzed and the fact that the scatter of the effective stresses can be assessed in terms of the standard deviation.

For the determination of the effective stresses in the individual numerical experiments, a strain energy based homogenization procedure is utilized which has been proposed earlier by the present authors for a finite strain analysis of strictly periodic cellular solids (Hohe and Becker [3]). Within this scheme, the mechanical response of a representative volume element for the given microstructure and a similar volume element consisting of the effective medium is defined to be macroscopically equivalent, if the average the strain energy density in both elements is equal provided that both volume elements are subjected to deformation states where the volume average of the deformation gradient is equal:

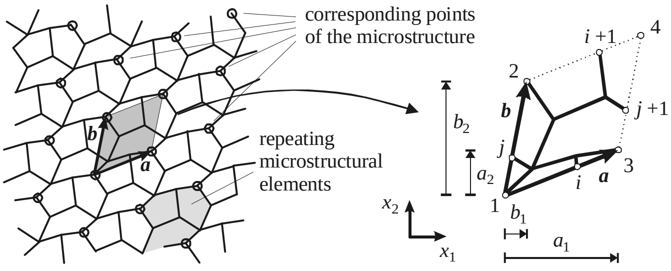


Fig. 1: RVE for general periodic 2D microstructures

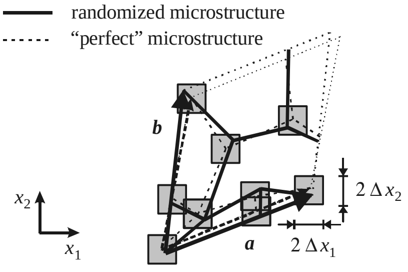


Fig. 2: Randomized RVE

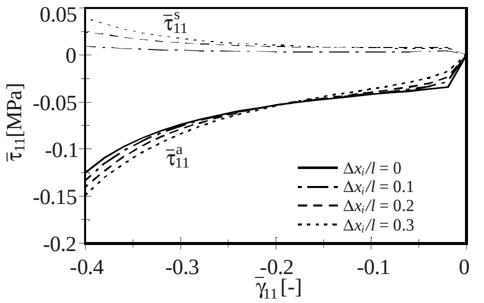


Fig. 3: Stress-strain response under uniaxial compression

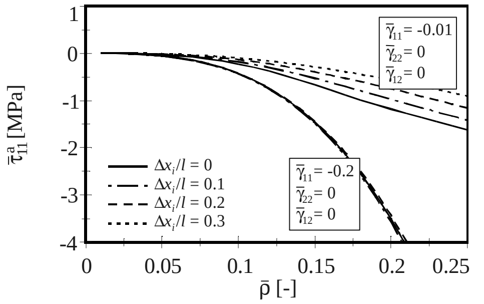


Fig. 4: Effect of the relative density (uniaxial compression)

$$\overline{w} = \frac{1}{V^{RVE}} \int_{\Omega^{RVE}} w\, dV = \frac{1}{V^{RVE*}} \int_{\Omega^{RVE}} w^* \, dV = \overline{w}^* \quad \text{if:} \quad \overline{F}_{ij} = \frac{1}{V^{RVE}} \int_{\Omega^{RVE}} F_{ij}\, dV = \frac{1}{V^{RVE*}} \int_{\Omega^{RVE}} F^*_{ij}\, dV = \overline{F}^*_{ij} \qquad (2)$$

The determination of the strain energy is performed numerically by means of a finite element model with periodic boundary conditions.

## RESULTS

The probabilistic homogenization scheme defined in the previous section is applied to the analysis of a two-dimensional model foam. As a periodic reference material, a regular hexagonal structure is used. Three different degrees $\Delta x_i / l$ of microstructural disorder are considered where $l$ is the nominal cell wall length for the periodic reference material. On the cell wall level, an Ogden-type hyperelastic constitutive model is employed. In a study of convergence a number of $n = 50$ numerical experiments proves to be sufficient.
The macroscopic stress-strain response in terms of the second Piola-Kirchhoff stress tensor as a function of the Green-Lagrange strain tensor is presented in Fig. 3. The considered example is a model foam with a relative density of $\rho = 5\%$ under uniaxial compressive deformation. For comparison, results obtained by means of a perfectly periodic model without any microstructural disorder are included as a solid line. Distinct effects of the microstructural disorder are observed for small effective strains where a kink in the stress-strain curve for the perfectly ordered microstructure indicates a bifurcation. No instabilities in the Eulerian sense occur for the geometrically imperfect disordered foams. At large effective strain levels, an increasing disorder results in increasing effective stresses. Throughout the considered effective strain range, increasing degrees of microstructural disorder yield increasing standard deviations indicating increasing scatter band widths, especially at large levels of the effective compressive strain.
In Fig. 4, the effect of the relative density is studied. Two different effective strain states are considered for which the relative density is varied from small values up to $\rho = 25\%$. For the large effective stress level ($\gamma_{11} = -0.2$), an approximately cubic dependence of the effective stresses on the relative density is obtained. In this case, the microstructure is in a postbuckling state where the microstructural mode of deformation is distributed bending of the entire cell walls in the model. Since the bending stiffness increases cubically with the cell wall thickness and therefore with the relative density, the observed cubic dependence of the effective stresses on the relative density is obtained. A similar behavior is observed for the small effective strain level at small relative densities. For large relative densities, the microstructure is in a prebuckling state due to the larger cell wall thicknesses. In this case the underlying microstructural mode of deformation changes from overall bending to longitudinal compression of the cell walls. Especially in this range, strong effects of the microstructural disorder are observed.

## CONCLUSION

Subject of the present study is a probabilistic homogenization scheme for hyperelastic cellular solids. Advantage of the direct probabilistic approach compared to previous studies is the possibility to address the scatter to be expected in the effective stresses and its numerical efficiency due to the employment of small-scale representative volume elements. It is observed that the microstructural disorder has distinct effects on the macroscopic stress-strain response of cellular solids. Therefore, classical deterministic approaches based on strictly periodic models might yield inaccurate results.

**References**

[1] Fortes, M.A., Ashby, M.F.: The effect of non-uniformity on the in-plane modulus of honeycombs. *Acta Mater.* **47**: 3469-3473, 1999.
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[3] Hohe, J., Becker, W.: Effective mechanical behavior of hyperelastic honeycombs and two-dimensional model foams at finite strain. *Int. J. Mech. Sci.* **45**: 891-913, 2003.