ON THE STABILITY OF OCEANIC VORTICES: A SOLUTION TO THE PROBLEM

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INTRODUCTION

The contradiction between observational and theoretical estimates of the lifespan of oceanic vortices (rings) has been identified more than two decades ago. Observations (e.g. Olson 1991) suggest that rings exist for years, whereas theoretical studies (see Benilov 2003 and references therein) indicate that they are unstable.

An attempt to resolve the contradiction has been made by Dewar & Killworth (1995), who considered a two-layer ocean with a Gaussian vortex in the upper layer and a weak co-rotating circulation in the lower layer ("deep flow"). It turned out that the latter could stabilize the vortex, or at least considerably reduce the growth rate of instability. This result has been later extended to other vortex shapes by Katsman *et al.* (2003): in all examples considered, a co-rotating deep flow reduced the growth rate (although no profile other than the Gaussian one would become entirely stable).

Before we describe the results of the present paper, note that Dewar & Killworth (1995) and Katsman *et al.* (2003) assumed that the deep circulation has the same shape as the upper-layer vortex. We shall use a different approach: the profile of the deep flow will be derived from the assumption that potential vorticity (PV) in the lower layer is constant. Such model is suggested by the fact that most oceanic vortices are shed by unstable frontal currents: when a vortex moves away from the current and arrives to a new location, the PV field below it cannot change and remains equal to its initial, background value. Observe also that the interface displacement, caused by the vortex, contributes to the lower-layer PV – hence, to maintain its initial value, the lower layer spins up. Remarkably, the resulting circulation is always *co-*, and never *counter-*, rotating!

The present paper examines, numerically and analytically, vortices in a two-layer ocean. It is demonstrated that vortices with uniform PV in the lower layer are baroclinically stable, regardless of their profiles in the upper layer.

FORMULATION

Consider a vortex in a two-layer ocean with rigid lid and flat bottom, with $H_{1,2}$ being the depths of the layers ($_1$ marks the upper layer). The vortex will be characterized by the swirl velocities, $V_{1,2}(r)$, where r is the radial coordinate. Various profiles of the upper-layer velocity $V_1(r)$ have been examined and yielded similar results – here, we shall present those for the Gaussian vortex,

$$V_1 = V_* \frac{r}{r_*} \exp\left(-\frac{r^2}{2r_*^2}\right),$$

where V_* and r_* are the amplitude and radius of the vortex. With regards to the lower layer, we shall consider two particular cases: compensated vortices, for which the lower layer is at rest, $V_2 = 0$; and vortices with uniform lower-layer PV, for which V_2 satisfies

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rV_2)\right] - \frac{f^2}{g'H_2}(V_2 - V_1) = 0 \tag{1}$$

(f is the Coriolis parameter and g' is the reduced gravity). Both types of vortices were examined for stability through the usual linear normal-mode analysis, which yields the following eigenvalue problem:

$$(cr - V_1)\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi_1}{dr}\right) - \frac{k^2}{r^2}\psi_1 - \frac{f^2}{g'H_1}(\psi_1 - \psi_2)\right] + Q_1'\psi_1 = 0, \tag{2}$$

$$(cr - V_2)\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi_2}{dr}\right) - \frac{k^2}{r^2}\psi_2 - \frac{f^2}{g'H_2}(\psi_2 - \psi_1)\right] + Q_2'\psi_2 = 0, \tag{3}$$

$$\psi_{1,2}(0) = \psi_{1,2}(\infty) = 0, \tag{4}$$

where $\psi_{1,2}(r)$ describe the spatial structure of the mode, k and c are the mode's azimuthal wavenumber and phase speed (if $\operatorname{Im} c > 0$, the vortex is unstable), and

$$Q_1' = \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rV_1)\right] - \frac{f^2}{g'H_1}(V_1 - V_2), \qquad Q_2' = \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rV_2)\right] - \frac{f^2}{g'H_2}(V_2 - V_1)$$

are the PV gradients (observe that equation (1) is equivalent to $Q_2' = 0$).

NUMERICAL RESULTS

First, the eigenvalue problem (2)-(4) was solved numerically. We computed the marginal stability curve on the (ε, r_0)-plane, where

$$\varepsilon = H_1/H_2, \qquad r_0 = r_*/R_d$$

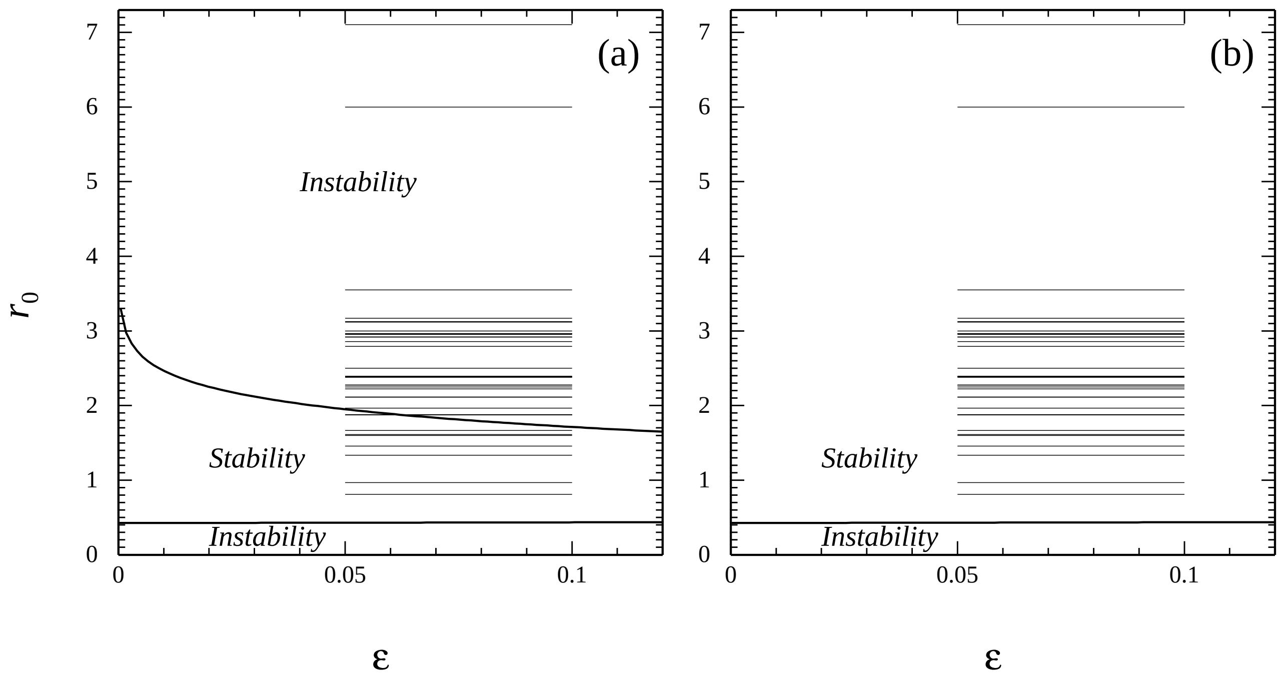

Figure 1. Stability of (a) compensated vortices, and (b) vortices with uniform PV in the lower layer, on the (ε, r_0)-plane (ε is the depth ratio of the ocean, r_0 is the nondimensional radius of the vortex). Horizontal segments show the rings catalogued by Olson (1991).

are the depth ratio of the ocean and nondimensional radius of the vortex ($R_d = \sqrt{g' H_1}/f$ is the Rossby radius).

The results for compensated vortices are shown in Fig. 1a. One can see that there are two regions of instability: vortices with small r_0 are unstable due to horizontal shear (*equivalent-barotropic* instability), while vortices with large r_0 are unstable with respect to vertical shear (*baroclinic* instability). We also show, on the same figure, the parameters of real-ocean rings catalogued by Olson (1991)[1]. One can see that only 7 out of Olson's 35 rings appear to be stable, which proportion is far too low to account for observations. It is also worth noting that none of the real rings is unstable barotropically, as their radii are two large for that.

The same computation has also been performed for vortices with uniform PV in the lower layer (see Fig. 1b). Remarkably, the entire region of baroclinic instability has disappeared, and all of Olson's (1991) rings are now stable.

ASYMPTOTIC RESULTS

The numerical results have been confirmed and complemented by an asymptotic analysis of the eigenvalue problem (2)-(4) under the assumption that the upper layer of the ocean is thin, i.e. $H_1 \ll H_2$. This assumption is not very restrictive as most oceanic rings are indeed localized in a thin layer (the only exceptions are the Southern Ocean rings, for which $H_1 \sim H_2$).

It has been shown that, if the "deep flow" satisfies (1), all vortices, not only the Gaussian ones, are baroclinically stable (for more details, see Benilov 2004).

CONCLUSION

We conclude that the model based on a vortex with uniform PV in the lower layer consistently describes the observed lifespans of oceanic rings.

References

[1] Benilov E.S.: Instability of quasigeostrophic vortices in a two-layer ocean with thin upper layer. *J. Fluid Mech.* **475**: 303–331, 2003.
[2] Benilov E.S.: Stability of vortices in a two-layer ocean with uniform potential vorticity in the lower layer. *To appear in J. Fluid Mech.*, 2004.
[3] Katsman C.A., Van der Vaart P.C.F., Dijkstra H.A., de Ruijter W.P.M.: On the stability of multi-layer ocean vortices: a parameter study including realistic Gulf Stream and Agulhas rings. *J. Phys. Oceanogr.* **33**: 1197–1218, 2003.
[4] Killworth P.D., Blundell J.R., Dewar W.K.: Primitive equation instability of wide oceanic rings. Part 1: linear theory. *J. Phys. Oceanogr.* **27**: 941–962, 1997.
[5] Olson D.B.: Rings in the ocean. *Ann. Rev. Earth Planet. Sci.* **19**: 283-311, 1991.

[1]Unfortunately, Olson (1991) presents no data on the thickness of the upper (active) layer of the ocean. Since most of oceanic vortices are localized in a thin layer, we assumed $0.05 < \varepsilon < 0.1$. As a result, vortices in Fig. 1 are represented by segments, not points.