XXI ICTAM, 15-21 August 2004, Warsaw, Poland

BOUNDARY LAYER DEVELOPMENT IN UNSTEADY FLOWS

Andrei Camyshev*, Andrei Kolyshkin*, Inta Volodko*

*Riga Technical University, Department of Engineering Mathematics, LV1048 Riga, Latvia

Summary The development of a boundary layer in unsteady viscous flows is analyzed in the present paper. Methods of linear and weakly nonlinear stability theory are used to investigate the stability characteristics of the flow.

INTRODUCTION

Rapidly changing unsteady flows are important in a wide range of applications. Examples include the design and analysis of water supply systems, flows in natural gas pipelines and flow of blood in arteries. Pump shutdowns or rapid changes in valve settings are known to generate unsteady flows in hydraulic devices or water supply systems. The pressure in unsteady flow can cause cavitation, pitting and corrosion [1]. An atherosclerosis plaque development in regions where shear stress changes direction can be triggered by blood flow unsteadiness [2]. In all these examples the flow characteristics (velocity and pressure) change considerably during short time intervals as a result of rapid deceleration or acceleration of the flow. In order to avoid possible problems associated with short time transients one needs to know the velocity, pressure and shear stress distribution in a channel or pipe.

In the present paper a mathematical model for the solution of rapidly changing unsteady flows is presented. The model is used to analyze the short time behaviour of viscous incompressible fluid subject to rapid deceleration. The paper is organized as follows. First, asymptotic solutions for unsteady laminar fluid flows in cylindrical channels of constant cross-section are derived. Three types of channels are considered: a plane channel, a pipe and an annulus. Flow region can be divided into two parts: a core region (where velocity distribution changes slowly) and boundary layer(s) near the wall where rapid changes are taking place. The method of matched asymptotic expansions is used to derive the solution and analyze the development of the boundary layer(s). Second, the velocity profiles of rapidly decelerating flows have inflection points and therefore are potentially highly unstable. Methods of linear stability theory are used to calculate stability characteristics of the flow under the quasi-steady assumption [3-6]. Hall and Parker [6] showed that the quasi-steady approach is justifiable if a perturbation grows faster than significant changes of the base flow can be observed. The quasi-steady assumption represents the first term of the asymptotic expansion of the WKB-type. It is shown that the critical Reynolds numbers decrease considerably as a result of rapid deceleration of the flow. The influence of the deceleration rate of the stability of the flow is investigated. Third, weakly nonlinear theory is applied in order to describe the evolution of the most unstable mode above the threshold. It is shown that the amplitude of the most unstable mode is governed by the complex Ginzburg-Landau equation.

MAIN RESULTS

The following model is proposed for the analysis of rapidly changing unsteady laminar flows. Consider an infinitely long horizontal cylinder of constant cross-section filled with a viscous incompressible fluid. The flow prior to acceleration or deceleration can be either steady or unsteady (but unidirectional). Starting from time $t = 0$ the flow is rapidly decelerated to zero (different types of channels and different transient scenarios are considered). Assume that the fluid flux through the cross-section of the channel has suddenly changed as a result of rapid deceleration. The change in velocity due to sudden change in pressure does not satisfy the no-slip condition at the wall of the channel (the process is initially inviscid). The sudden pressure change generates additional vorticity near the wall which starts to diffuse towards the core region of the channel. A boundary layer starts to develop near the wall as a result of diffusion. Thus, it is natural to divide the flow region into the (inviscid) core region (where the velocity distribution changes slowly) and the boundary layer(s) where rapid changes of the flow characteristics are taking place.

The method of matched asymptotic expansions is used to construct the solution. In particular, the core region is represented by the outer part of the expansion and the boundary layer is described by the inner part of the expansion. Excellent agreement between exact and asymptotic solutions for the case of a sudden blockage of a pipe is found for short time intervals. Calculations show that the velocity profiles of rapidly decelerating flows have regions of reverse flow with inflection points. Thus, these profiles are potentially highly unstable. Linear stability analysis is used to obtain the stability characteristics of the flow. The stability analysis is performed under the quasi-steady assumption. In other words, it is assumed that the rate of change of the base flow with respect to time is smaller than the growth rate of perturbations. The validity of the quasi-steady assumption for the analysis of rapidly changing unsteady flows in circular pipes is assessed in [3,4].

The linear stability problem is solved by a pseudospectral collocation method based on Chebyshev polynomials. Calculations show that the quasi-steady assumption gives reasonable results for rapidly decelerating laminar and turbulent flows in a pipe. Linear stability analysis for all three types of channels considered in the paper shows that the development of instability depends on the relative importance of two factors: the movement of the inflection point

towards the centre of the channel and the reduction of the difference between the largest and smallest values of the velocity as time increases. The critical Reynolds numbers decrease considerably during short time interval after deceleration. Both symmetric and asymmetric modes are studied for channels with cylindrical symmetry (circular pipe or annulus). It is shown that depending on the deceleration scenario either symmetric or asymmetric modes are the most unstable modes.
The results of linear stability calculations are compared with experimental data [5]. The results are found to be in qualitative agreement with experimental data in [5] (in terms of the pattern of the most unstable mode). The experimentally observed wavelengths of the most unstable mode are close to the values obtained by means of linear stability analysis.
Linear stability theory can be used to determine when does a particular flow become unstable. Methods of linear theory can be used to describe the structure of the critical motion above the threshold. However, linear theory cannot predict the evolution of the most unstable mode. Weakly nonlinear theories [7] are used to derive an evolution equation for the most unstable mode. The Ginzburg-Landau equation is one of the popular equations which is used to study nonlinear dynamics of the flow. Note that the Ginzburg_Landau equation is used in the literature in two ways: first, as a phenomenological equation and second, as an equation which appears in many hydrodynamic stability problems when weakly nonlinear analysis is applied. In the present paper weakly nonlinear theory is used to analyze the effect of nonlinearities analytically. It is shown in the paper that the amplitude of the most unstable mode above the threshold is governed by the complex Ginzburg-Landau equation. That is, the paper shows that the Ginzburg-Landau equation does not have to be assumed; it can be derived from the Navier-Stokes equations. The coefficients of the equation are evaluated numerically for several particular cases, namely, for the case of rapidly decelerating flow in a pipe and plane channel.
The Ginzburg-Landau model is compared with experimental data found in [5]. Some of the experiments in [5] indicate the presence of secondary vortices after the base flow loses stability. We calculated the coefficients of the Ginzburg-Landau equation for these cases. It is found that for all the cases where secondary motions were observed in experiments, the Landau constant was positive. Thus, the existence of a finite equilibrium state is also confirmed by the Ginzburg-Landau equation.

CONCLUSIONS

The paper analyzes the development of a boundary layer in rapidly decelerating viscous flows in channels of constant cross-section. The initial development of the boundary layer is described by the method of matched asymptotic expansions. The velocity profiles are found to contain inflection points. Linear stability theory is used to analyze the behaviour of the flow for short time intervals. It is found that the critical Reynolds numbers of the flow decrease as a result of rapid deceleration. The values of the critical Reynolds numbers depend on the deceleration rate. The shorter is the deceleration time, the greater is the decrease of the critical Reynolds numbers. Methods of weakly nonlinear theory are used to analyze the development of instability above the threshold. It is found that the amplitude of the most unstable mode is governed by the complex Ginzburg-Landau equation. The results are compared with available experimental data in the literature. Reasonable agreement between the experimental and theoretical data is found.

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