

## ON THE CONTACT THERMOELASTIC PROBLEM WITH FRICTIONAL HEATING, WEAR AND AUTO-VIBRATIONS

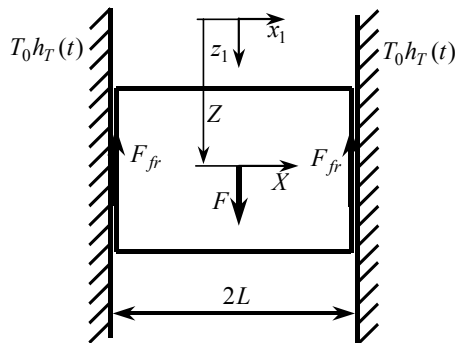
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**Summary** The solution of a contact thermoelastic problem of a solid in the form of an elastic layer moving between two rigid walls with the heat generation and wear is presented. It is assumed that the friction coefficient depends on the relative velocity between the contacting bodies. A stability of the stationary solution is studied. Computation of contact parameters during heating of the bodies is performed.

### PROBLEM FORMULATION

Friction, wear, heat generation, relative velocity and temperature deformation are complex processes which influence each other making up a sole diverse process of a friction unit work [1-4]. Consider contact and wear of one-dimensional model of the thermo-elastic contact of a body with a surrounding medium. Assume, that this body is represented by a rectangular plate (Fig.1). The vector components related to displacements as the plate temperature do not depend on  $Z$ , and the non-zero displacements  $U(X,t), W(X,t)$  and temperature  $T(X,t)$  components depended only on the co-ordinate  $X$  and time  $t$ . The plate has the mass  $M_1$  subject to the force  $F = F_* h_T(t)$  and moves vertically along walls in direction  $z_1$  of the rectangular co-ordinates  $0x_1y_1z_1$ . The distance between walls is always equal to initial plate thickness  $2L$ . It is assumed also that according to Coulomb's law the friction force is proportional to normal part of reaction and kinematical friction coefficient  $f$ . The friction coefficient is assumed to be a nonlinear function of a relative velocity [2,3,5,6]. It is assumed that the heat conduction between the layer and the walls temperature is governed by the formula  $T_0 h_T(t)$  ( $h_T(t) \rightarrow 1, t \rightarrow \infty$ ). It



causes a parallelepiped heat extension in the direction of  $0x_1$ , and the body starts to contact with walls. In the result of this process a frictional contact and wear on the parallelepiped sides  $X = \pm L$  occurs. We assume Archard's law of wear [1], a speed of plate wear is proportional to the power of frictional force.

### RESULTS AND DISCUSSION

From the mathematical viewpoint our problem is reduced to solution of the following dimensionless form of the non-linear system of integral and differential equations:

$$p(\tau) = Bi \int_0^\tau \dot{h}_T(\xi) G_p(\tau - \xi) d\xi + \gamma \int_0^\tau f(\dot{z}) \dot{z}(\xi) p(\xi) \dot{G}_p(\tau - \xi) d\xi - u^w(\tau), \quad (1)$$

$$\dot{z}(\tau) = \varepsilon_1 \left[ m_0 \int_0^\tau h_F(\xi) d\xi - \int_0^\tau f(\dot{z}) p(\xi) d\xi \right], \quad \dot{u}^w(\tau) = k^w \dot{z}(\tau) p(\tau) \quad (2)$$

which yields the non-dimensional pressure  $p(\tau)$  and velocity  $\dot{z}(\tau)$ . The temperature is defined through the following formula

$$\theta(x, \tau) = Bi \int_0^\tau \dot{h}_T(\xi) G_\theta(x, \tau - \xi) d\xi + \gamma \int_0^\tau f(\dot{z}) \dot{z}(\xi) p(\xi) \dot{G}_\theta(x, \tau - \xi) d\xi, \quad (3)$$

$$\{G_p(\tau), G_\theta(l, \tau)\} = \frac{1}{Bi} - \sum_{m=1}^{\infty} \frac{\{2Bi, 2\mu_m^2\}}{\mu_m^2 [Bi(Bi+1) + \mu_m^2]} e^{-\mu_m^2 \tau}, \quad (4)$$

and  $\mu_m$  are the roots of the following characteristic equation  $\text{tg } \mu_m = Bi/\mu_m, m = 1, 2, \dots$

The numerical analysis of the problem is performed using the Runge-Kutta method.

In this case the steel made parallelepiped plate with  $L = 0.01$  m,  $T_0 = 5^\circ C, z^\circ = \dot{z}^\circ = 0$  and with non-constant friction coefficient is studied. In order to confirm the given conclusions, numerical analysis is carried out for  $Bi = 5$  (critical value  $\varepsilon_1 \approx \tilde{\varepsilon} = 586.5$ ), and the computational results are carried out for a few values of the parameter  $\varepsilon_1 = 400; 800$ .

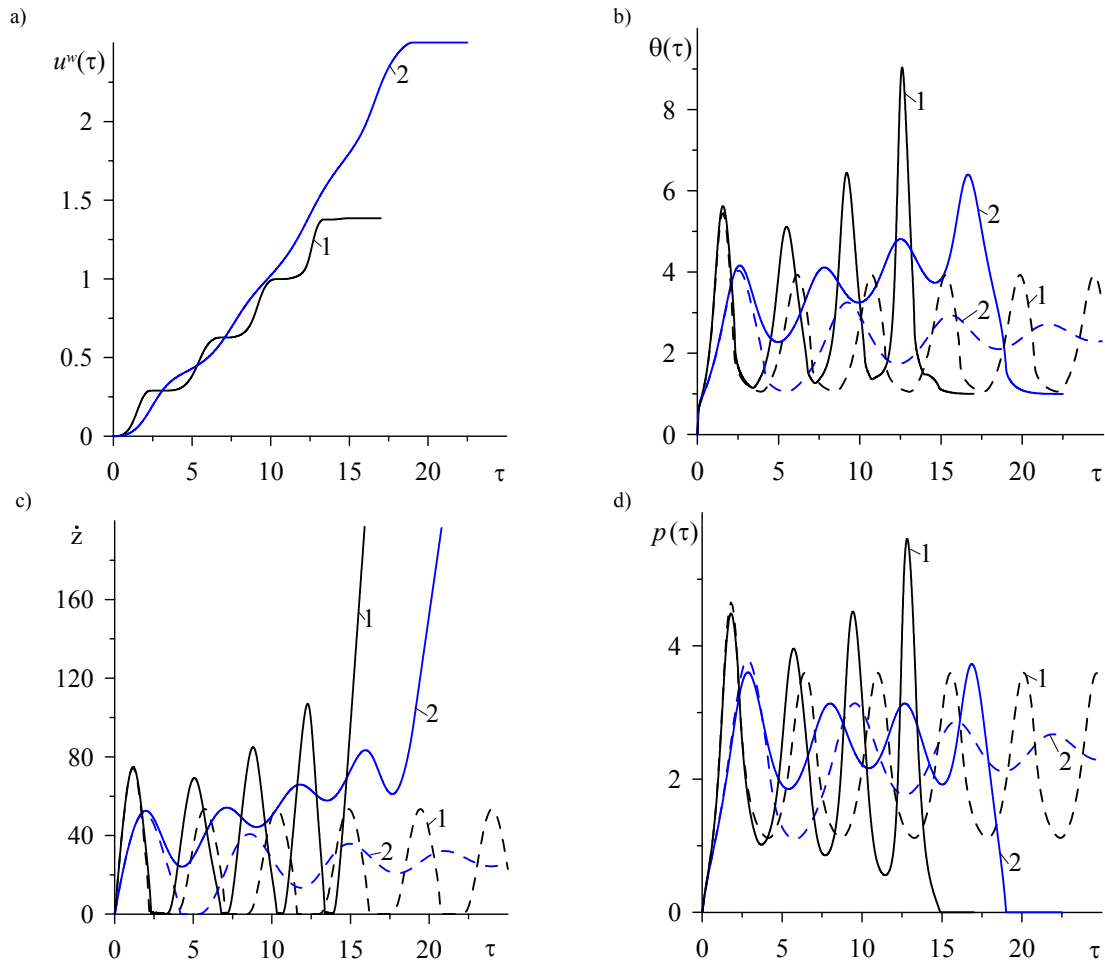


Figure 2 Time history of non-dimensional wear (a), contact temperature (b), body's velocity (c) and contact pressure (d) for various values of  $\varepsilon_1$ , solid curves  $k^w = 0.001$ , dashed curves  $k^w = 0$  (curves 1:  $\varepsilon_1 = 800$ ; curves 2:  $\varepsilon_1 = 400$ )

Dependence of non-dimensional wear  $u^w(\tau)$ , contact temperature  $\theta(\tau) \equiv \theta(\pm 1, \tau)$ , body velocity  $\dot{z}(\tau)$  and contact pressure  $p(\tau)$  on the non-dimensional time  $\tau$  are also analysed. For  $\varepsilon_1 = 800$ ,  $k^w = 0$  a stationary solution is unstable, and a limiting stick-slip cycle appears with the expected period  $T = 4.18$  (frictional auto-vibrations [5,6]). For  $k^w = 0.001$  wear occurs and the sliding contact is lost.

## CONCLUSIONS

In this work the results devoted to a novel problem of the mechanical system exhibiting frictional thermoelastic contact of a moving body subject to both non-constant friction coefficients and wear are presented and discussed. It is worth noticing that in the case of non-constant friction coefficient and heating, the self-excited vibration can appear in our system without an elastic part (stiffness). The last phenomenon is caused by body heating while accelerating, friction increase, and than braking and cooling of the system. Owing to wear auto-vibrations are damped, and the contact system state is bounded.

## References

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