THERMOCAPILLARY CONVECTION IN CYLINDRICAL GEOMETRY

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Summary Thermocapillary convection in two types of cylindrical geometry is investigated in two- and three-dimensional numerical simulations: a liquid bridge heated from the upper wall and an open annulus heated from the outside wall. For the parameter ranges considered, it is found that dynamic free-surface deformations are negligible and do not induce transitions to oscillatory convection in axisymmetric models. Moreover, only steady convection is possible at any Reynolds number (Re) in strictly axisymmetric computations. In our three-dimensional models, the nondeformable free surfaces are either flat or curved as determined by the fluid volume (V) and the Young-Laplace equation. Convection is steady and axisymmetric at sufficiently low values of Re with either nondeformable or deformable surfaces. Transition to oscillatory three-dimensional motions occurs as Re increases beyond a critical value dependent on the aspect ratio, the Prandtl number and V. Good agreement with available experiments is achieved in all cases.

INTRODUCTION

The most popular methods for growing crystals are the Czochralski and floating zone techniques. In the present paper, we consider thermocapillary convection in a pair of relevant cylindrical geometry: a liquid bridge in a half-zone model of the float-zone crystal-growth process as shown in Fig. 1(a), and an open cylindrical annulus heated from the outside wall as shown in Fig. 1(b) constituting a model for the Czochralski crystal growth system. Thermocapillary convection with undeformable flat or curved surfaces is investigated in two- and three-dimensional numerical simulations, and critical conditions for transition to oscillatory states are established. Dynamic free-surface deformations are discussed only in the axisymmetric numerical simulations.

MATHEMATICAL AND NUMERICAL MODELS

Surface tension is assumed a linear function of temperature $\sigma = \sigma_0 - \gamma(T - T_0)$ where $\gamma = -\partial\sigma / \partial T$, and subscript 0 represents a reference state. Neglecting body forces, the non-dimensional governing equations are as follows:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\mathrm{Re}\left(\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v}\mathbf{v})\right) = -\nabla P + \nabla^2 \mathbf{v} \tag{2}$$

$$Ma\left(\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{v}T)\right) = \nabla^2 T \tag{3}$$

where **v**, P and T are respectively the non-dimensional velocity vector, pressure and temperature. Re is the Reynolds number, Pr is the Prandtl number, and Ma is the Marangoni number.

In order to numerically solve the problem with either undeformable or deformable surfaces, the governing equations and boundary conditions are transformed from the physical domain into a rectangular computational domain. The transformed equations and boundary conditions are solved by a finite volume method employing a SIMPLER algorithm.

RESULTS AND DISCUSSION

Thermocapillary convection in liquid bridges

We have investigated thermocapillary convection up to Re=5000 with Pr=1, Ar (aspect ratio) =1, V=1, Bi (Biot number) =0 and capillary numbers (Ca < 0.1), and have found no oscillatory axisymmetric states in liquid bridges with either nondeformable or deformable free surfaces. Assuming non-deformable flat interfaces with Pr=1 and Ar=1, the critical Re, Re_c, for transition to oscillatory states is about 2500 from linear theory and three-dimensional numerical simulations. Since Ca is in the range $O(10^{-2})$-$O(10^{-3})$ in most experiments, we conclude that dynamic free-surface deformations do not induce transition to oscillatory, axisymmetric convection. Thus only azimuthal waves can generate oscillations in a liquid bridge with either nondeformable or deformable free surfaces. Fig. 2 shows two-dimensional free surfaces with Pr=1, Ar=1, Bi=0, Ca=0.05 and various Re. The free surfaces are convex near the lower cold wall, and change from concave to convex near the hot wall with increasing Re because of stronger return-flow in the interior. At sufficiently low Re, the free surface is almost asymmetric about its mid point. It has two peaks and is elevated near the cold stagnation point where surface pressure achieves its maximum value. As Re increases at fixed Ca, the free surface develops three peeks. The extremely small magnitude of the deformations should be noted.

In three-dimensional models with Ar=0.714, Pr=27, Re_c for onset of oscillations with V=1.138 (convex), 1 (flat) and 0.755 (concave) are about 210, 210 and 220, respectively. The temperature fluctuations with V=1 consist of a hot and a cold spot rotating clockwise, i.e m (wavenumber) =1. This rotating pattern with m=1 remains unchanged with increasing Re. Two pairs of hot and cold spots, i.e. a wavenumber of 2, are rotating clockwise with V < 1.

Thermocapillary convection in open annuli

Thermocapillary convection up to Re=5000 with Pr=30, Ar=1, Bi=0 and Ca=0.1 was computed and no axisymmetric oscillatory states with either nondeformable or deformable surfaces were found. In three-dimensional numerical simulations with Pr=30, Ar=1, Bi=0 and Ca=0 (nondeformable flat surface), Re_c was about 2200. Thus dynamic free-surface deformations do not induce transition to unsteady, oscillatory axisymmetric convection. Fig. 3 shows free surfaces with Ca=0.05 and various Re in axisymmetric annuli heated from the inside wall. The surface is convex near the cold wall where surface pressure has a maximum positive value at the stagnation point, and concave near the hot wall. Two peaks appear at the free surface at sufficiently low Re. As Re increases, additional ripples occur at the free surface which can be convex close to the hot corner. Surface elevations and depressions decrease with increasing Re due to this change in topology, volume conservation and curvature. Again, we observe extremely small surface deflections.

Re_c for onset of oscillations in three-dimensional annuli with Bi=0, Pr=6.84 and Ar=1, 2.5, 3.33 and 8 are about 740, 490, 490 and 560, respectively. With Ar=1, 2.5 and 3.33, we observe 5, 9 and 12 azimuthal wavetrains, respectively, rotating clockwise at the free surface near the critical Re. Twenty pulsating azimuthal wavetrains with Ar=8 are found on the free surface. The inside wall looks like the source of the waves: the waves are generated at the inside cold wall and travel to the outside hot wall. We thus have travelling r-waves and pulsating θ-waves in cylindrical-shallow liquid layers. The critical wavelength λ=2.5 (Pr = 6.84) from linear theory implies a wave number of 20 at the inside wall, which is in good agreement with our numerical result.

CONCLUSIONS

Thermocapillary convection in liquid bridges and open cylindrical annuli is investigated in two- and three-dimensional numerical models. For the parameter ranges considered, dynamic free-surface deformations are negligible and do not induce transitions to oscillatory convection in axisymmetric models. Thus only steady convection is possible at any Re in strictly axisymmetric computations. In the three-dimensional models, the nondeformable free surfaces are either flat or curved as determined by the fluid volume, and the Young-Laplace equation. Convection is steady and axisymmetric at sufficiently low values of Re with either nondeformable or deformable surfaces. Transition to oscillatory three-dimensional motions occurs as Re increases beyond a critical value dependent on the aspect ratio, the Prandtl number and V. Good agreement with available experiments is achieved in all cases.

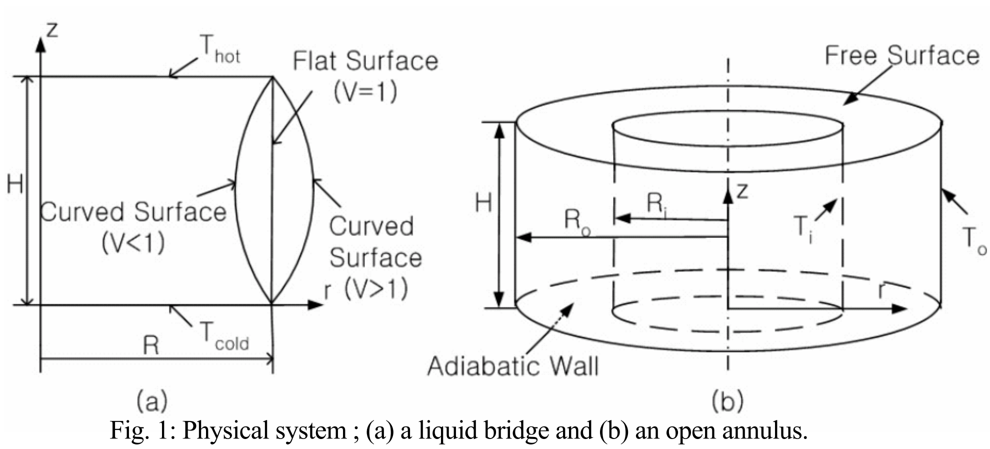

Fig. 1: Physical system ; (a) a liquid bridge and (b) an open annulus.

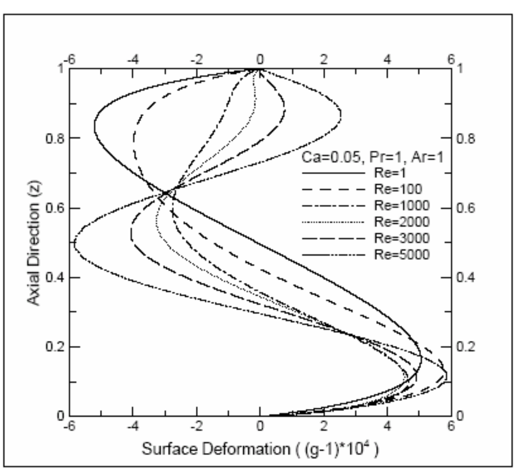

Fig. 2: Free surface deformations in liquid bridges.

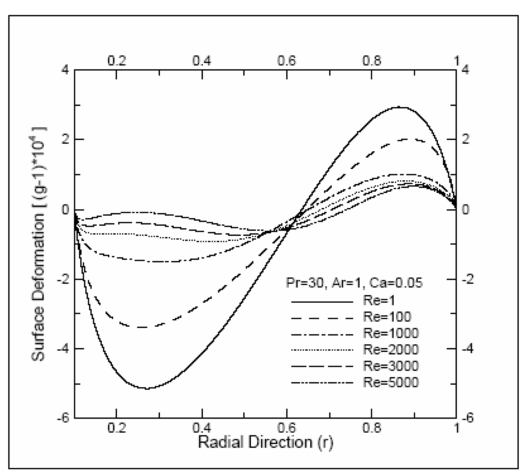

Fig. 3: Free surface deformations in open annuli.