DEVELOPMENT AND PRACTICAL APPLICATION OF WENO SCHEMES FOR COMPRESSIBLE FLUID FLOW COMPUTATIONS

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Summary Application of high-order shock-capturing schemes to numerical simulation of problems in supersonic aerodynamics is considered. Euler and Navier-Stokes solvers based on employing modern essentially non-oscillatory (ENO) and weighted essentially non-oscillatory (WENO) schemes are described. Examples of numerical simulations of 2D and 3D shock-dominated flows with high-order schemes are given. For a number of problems, high-quality numerical schlieren visualizations and interferograms are compared with experimental patterns. It is demonstrated that WENO schemes can be considered as a very promising candidate for Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) of high-speed compressible fluid flows.

MOTIVATION

The spectral and high-order finite-difference methods are succesfully used now for simulation of incompressible and subsonic compressible flows. They are inappropriate, however, for flows with shocks and other flow disconinuites due to the absence of shock-capturing properties. On the other side, the standard, second order, shock-capturing total variation diminishing (TVD) schemes are optimally suited for simulations of supersonic flows with a small number of isolated shock waves. However, at local extrema of the solution, they are reduced to first-order schemes and pollute the solution by excessive numerical diffusion. Therefore, problems containing both shocks and a large number of complex structures in the region of smoothness require more precise numerical tools. This need is especially evident for such applications as Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) of compressible transitional and turbulent flows, simulation of separated and jet flows, computational aeroacoustics, simulation of supersonic combustion and detonation, and many others.

The present paper is aimed at development of algorithms and codes that can robustly capture shocks and, simultaneously, accurately simulate the smooth part of flows, which contain complicated shock/shock, shock/vortex, shock/instability wave interactions. The modern ENO and WENO schemes [1] are natural candidates for the role of a basic numerical tool in these algorithms and codes. We describe Euler and Navier-Stokes solvers based on WENO schemes and give examples of their application to numerical simulation of complicated, 2D and 3D, shock-dominated flows.

DEVELOPMENT OF ALGORITHMS AND SOLVERS

Though high-order ENO and WENO schemes are proved to be both high-accurate and efficient in numerical sumulations of simple test problems, there are some difficulties in their application to more complicated flows of practical importance. In particular, finite-volume (FV) high-order schemes are very expensive for multidimensional problems while finite-difference (FD) ENO/WENO schemes suffer from violation of the so-called geometrical conservation law and do not conserve exactly a uniform flow. It hinders simulation of problems with complex geometry. In fact, FV WENO schemes are expensive because their numerical fluxes are not pointwise ones and should be calculated by integration along a cell face. Also, a correct, high-order, treatment of curvilinear grids with FD WENO schemes is difficult since their numerical fluxes approximate the exact pointwise fluxes only with the 2nd order. A new FD WENO scheme using only pointwise values of the conservative variables and fluxes, which are high-order approximations of exact quantities, is constructed. For curvilinear grids, it has the property of exact conservation of the uniform flow.

Further, a 4th-order approximation of the viscous terms on a compact 5-point (in each direction) stencil is introduced, which is coupled in a natural way with the new WENO scheme when solving the Navier-Stokes equations.

EXAMPLES OF NUMERICAL SIMULATIONS

A number of numerical simulations of 2D and 3D compressible flows containing complex configurations of shock waves and other flow discontinuities have been performed with high-order WENO schemes. They include shock wave propagation and diffraction, shock/shock and shock/boundary layer interaction, shock-wave structure of imperfectly expanded supersonic jets, development of hydrodynamic instability and transition to turbulence in high-speed free shear flows, etc. Below two examples of computations are given.

Figure 1 shows the shock wave diffraction on a triangular body at the shock wave Mach number $\mathrm{M}_s = 1.3$. A comparison of numerical and experimental pictures (the latter is taken from [2]) reveals that numerical simulation reproduces very accurately all features of the flow structure including such delicate details as the tiny secondary Mach stem, the chain of vortices near the trailing edges, the double slip lines linking the triple points with the main vortices, etc.

The second example is LES of a plane underexpanded jet at the jet Mach number $\mathrm{M}_j = 1.5$ and the jet/ambient pressure ratio $p_j/p_a = 2$. Flow turbulence with a prescribed spectrum is used as inflow forcing. Instanteneous vorticity isosurfaces are shown in Fig. 2. For these flow conditions, approximately two barrel-like jet cells develop in the computational domain. Accelerations and decelerations of the jet flow owing to the system of expansion and compression waves have

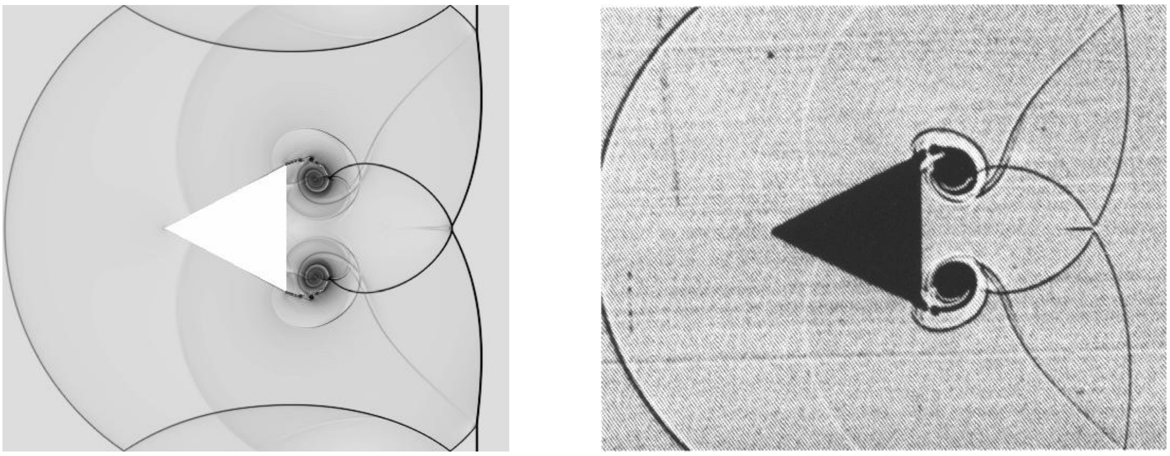

Figure 1. Numerical schlieren picture (left) and experimental shadowgraph (right) of shock wave diffraction on a finite wedge.

a great effect on turbulence characteristics. The expansion waves cause decreases in the turbulence level, whereas the compression results in a sharp increase in the turbulent fluctuations. This is illustrated by Fig. 3, where the time averaged r.m.s. streamwise velocity fluctuations in the central plane are given versus the downstream distance. The sharp peaks in the distribution correspond to the locations where the shock wave reflects from the central plane.

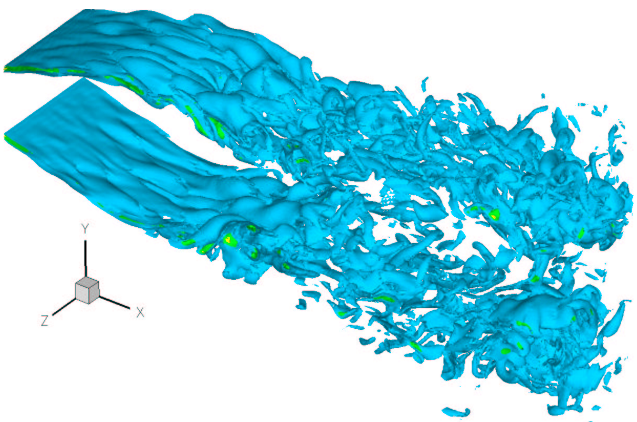

Figure 2. Instanteneous vorticity isosurfaces of LES of a plane underexpanded jet.

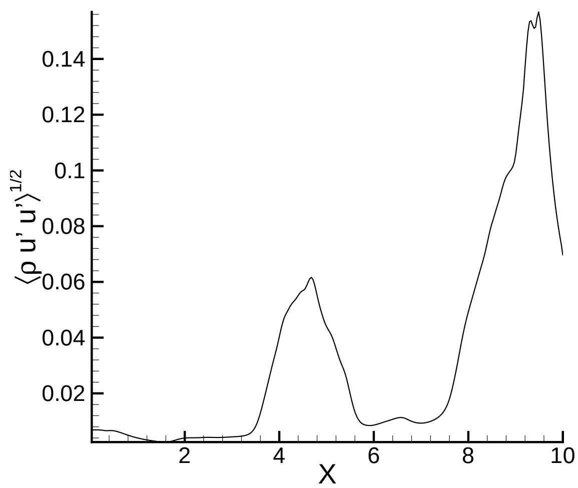

Figure 3. Distribution of r.m.s. velocity fluctuations along the jet centerline.

CONCLUSIONS

Computations performed for a number of different problems confirm that high-order WENO schemes are powerful tools for simulation of compressible fluid flows. They can be considered as very promising candidates for DNS and LES of turbulent supersonic flows.

References

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