IMPERFECTION SENSITIVITY OF CIRCULAR ARCH'S NON-LINEAR MODES

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Summary The paper addresses the imperfection sensitivity of non-linear vibration modes of moderately low *and* slender circular arches. Such structures may undergo unstable symmetric bifurcation prior to snap-through, under static uniform radial loading. Such instability corresponds to a buckling load that happens to be imperfection sensitive. A small imperfection may drastically cause the reduction of the arch's critical load and, consequently, of its stiffness and vibration properties.

INTRODUCTION

The non-linear static and dynamic behaviour of circular arches is strongly dependent of the slenderness ratio $\left(\lambda = \alpha^2 R / h\right)$, where α, R and h stand for the arch's semi-central angle, radius and thickness, respectively. It is well known that very low arches may experience limit-point instability followed by snap-through. It is also known that arches that are moderately low and slender ($\lambda > 2.65$), under static uniform radial loading, may experience unstable symmetric bifurcation prior to snap-through. Such instability corresponds to a buckling load that happens to be imperfection sensitive. In other words, a small imperfection (such as deviation in geometry, load offset, initial stresses, etc) may drastically cause the reduction of the arch's critical load and, consequently, of its stiffness and vibration properties. The present investigation addresses the issue of imperfection sensitivity of non-linear vibration modes of moderately low and slender circular arches.

NON-LINEAR MODES OF VIBRATION

With regard to non-linear modes, it is mandatory to refer to the recent and relevant work of Shaw and co-workers [1], which set the foundations for evaluating non-linear modes of systems with few degrees of freedom by the invariant manifold technique. The authors have broadened this technique to encompass the analysis of finite-element models of plane frames [2]. Further, they have developed another technique, based upon the method of multiple scales [3], which proved to be capable of handling this problem and, even more, the one of evaluating the so-called non-linear multimodes, which come into play when the system is under internal resonance due to strong modal coupling. It is believed that both techniques, within the frame of finite-element modelling, are state-of-the-art.

PARAMETRIC ANALYSIS AND ASSESSMENT OF IMPERFECTION SENSITIVITY

An extensive parametric survey is under way to consider different slenderness ratios, as well as varied static load levels and equivalent imperfection parameters, so as to supply a sharp picture of non-linear free vibrations in circular arches. Planar frame finite-element models are proposed to represent circular arches, so that it is possible to make use of the computational codes already developed at the Escola Politécnica's Computational Mechanics Laboratory. Of course, prior to any non-linear analysis, a usual linear undamped modal analysis is performed for reference. Next, non-linear normal modes are evaluated following the two approaches afore mentioned, namely the invariant manifold and the multiple-scale techniques, taking into account viscous damping, if so wished, yet still considering the vibrations about the undeformed configuration, and results are carefully compared with each other. Then, for a range of the static loading and the equivalent imperfection parameter, a non-linear static analysis is performed, so as to render the deformed equilibrium configuration. Finally, the non-linear vibration modes about the deformed equilibrium configuration are searched by the multiple-scale technique and the imperfection sensitivity is established.

RESULTS

At present, the authors have already been able to draw results for all types of proposed analyses, yet for only one set of parameters (slenderness ratio, static uniform radial loading, vibration amplitude and imperfection parameter). These first results confirm the authors' expectation with respect to the importance of considering non-linear analyses to establish the vibration properties of moderately low and slender arches. More results will be available at short notice, as an extensive parametric survey continues to be carried out according to the same methodology, thus allowing for a broader view of the problem. Just for the sake of presenting a case study, a pinned-pinned arch (Young's modulus $E = 73.3$ GPa; semi-central angle $\alpha = 0.894$ rad; radius $R = 1$ m; section height $h = 0.002$ m; section depth

$b = 0.02$ m) has been considered. It has been discretised with 14 Bernoulli-Euler beam-elements and its first non-linear mode has been evaluated by both techniques (invariant manifold and the method of multiple scales). Only undamped analyses have been considered — although the developed procedures can also handle systems with viscous-damping —, so that the free vibration responses will last longer and the comparison between the different analyses can be assessed along an extended time range. The unloaded structure without imperfection has been analysed in the first simulation. The rotation at the left-hand side support p_1 has been taken as modal variable. Figure 1 presents the comparison between the linear and non-linear responses. The non-linear responses were obtained by the multiple scale method (ms) and the invariant manifold technique (im). As it is seen in Figure 1, there is a perfect fitting between both non-linear solutions, yet a clear distinction between them and the linear solution as time progresses, due to small frequency differences. Next, the structure without imperfection has been subjected to a static uniform radial loading of $p = 9$ N/m (approximately 75% of the critical load [4], $p_{cr} = 12.1$ N/m), prior to be forced to vibrate freely. Figure 2 shows the time-evolution of p_1 (about the equilibrium configuration) for this situation. Then, an imperfection has been introduced into the system, in the form of an anti-symmetric loading superimposed to the static uniform radial loading. An equivalent imperfection parameter ε can be conveniently introduced to give a measure of the anti-symmetric loading with respect to the symmetric one. In the numerical essay it has been considered $\varepsilon = 8.8\%$. Again, the first non-linear mode about the new deformed configuration has been evaluated and the time-evolution of p_1 for this new situation is also depicted in Figure 2. A comprehensive parametric survey must also take into account that the non-linear modes are characterised by a non-linear relationship between frequency and amplitude. For the imperfect system, it is necessary to keep the amplitudes small; otherwise strong cubic non-linearities will spoil the solution. The linear theory first natural frequency is found to be 4.84 Hz; the first non-linear mode frequency about the undeformed equilibrium configuration is 4.76 Hz, as evaluated by both the invariant manifold and the multiple scales techniques; the first non-linear mode frequency about the deformed equilibrium configuration of the perfect system, under a static uniform radial loading of 75% of the critical one, is 2.38 Hz, as evaluated by the multiple-scale technique; the first non-linear mode frequency about the deformed equilibrium configuration of the imperfect system, under a static uniform radial loading of 75% of the critical one plus an anti-symmetric loading of 8.8% of the symmetric one, is 2.31 Hz, as evaluated by the multiple-scale technique. For the ensemble of control parameters considered here, it is seen that the static uniform radial loading represents a major influence upon the free-vibration characteristics of moderately low and slender circular arches (51% decrease in the frequency with respect to the linear theory). Imperfection sensitivity is only apparently mild here, responding for a "modest" 3% additional reduction with respect to the non-linear mode of the perfect system (yet for extremely small vibration amplitudes 10^{-5} rad!). Nevertheless, the system becomes extremely sensitive to the initial conditions. Larger vibration amplitudes would produce severe change in the vibration properties, which cannot be properly foreseen by the technique. As pointed out before, a complete picture on imperfection sensitivity will only be available after completion of an exhaustive parametric survey, considering a full range of slenderness ratios, static loading levels with respect to the critical one, equivalent imperfection parameters and vibration amplitudes.

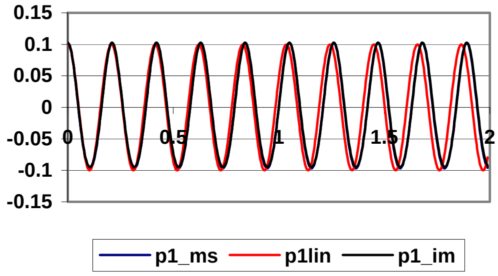

Figure 1: Time evolution of rotation p_1 with respect to the undeformed configuration

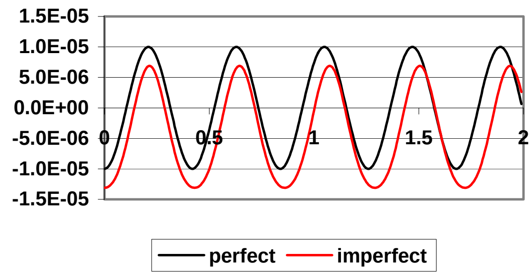

Figure 2: Time evolution of rotation difference Δp_1 with respect to the equilibrium configuration

References

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