CHAOS IN WAVE FRONT PROPAGATION IN HETEROGENEOUS MEDIA

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Summary The appearing of the determined chaos at propagation of nonstationary volume and surface waves in heterogeneous media is investigation. The waves represent fronts, on which stresses have a discontinuity. The conditions which satisfy distributions of the determined heterogeneity and appear stochastic states in behaviour of rays are obtained. A changes of intensity of discontinuity and geometrical parameters of wave fronts take place along trajectories of rays. Two different type of going to the determined chaos in dynamic of rays.

OBTANING OF THE EQUATIONS FOR INTENSITIES, PARAMETERS OF INTERIOR GEOMETRY OF FRONTS AND RAYS

The propagation of nonstationary waves in an isotropic heterogeneous elastic medium is describing by the equations

$$\nabla_j \sigma_{ij} - \rho(x)\ddot{u}_i = 0, \quad i, j = 1, 2, 3 \tag{1}$$

$$\sigma_{ij} = \lambda \nabla_k u^k G_{ij} + \mu(\nabla_i u_j + \nabla_j u_i) \tag{2}$$

where G_{ij} - metric tensor of space for volume waves and free surface for surface waves, λ, μ are elastic modulus.

Conditions on a free surface S

$$\sigma_{ij} n_j = 0, \; x^n \in S. \tag{3}$$

The investigation of the determined chaos for system of equations (1) in partial derivatives is possible to do going to the ordinary differential equations with the help of dynamic, kinematic, geometrical conditions of compability at the front waves and Fermat's principle.

The obtained closed systems of the nonlinear differential equations for volume waves have the form

$$\frac{d\omega}{ds} - \left(\Omega - \frac{1}{2}\cdot\frac{d\ln c}{ds}\right)\omega = 0, \; c = \sqrt{\frac{\Lambda_\alpha}{\rho}}, \;\; \Lambda_l = \lambda + 2\mu, \;\; \Lambda_t = \mu \;\; , \;\; \alpha = l, t. \tag{4}$$

$$\frac{d\Omega}{ds} = 2\Omega^2 - \mathrm{K} - \frac{c_{,\alpha\beta}\, g^{\alpha\beta}}{2c}, \;\; \frac{dK}{ds} = 2\Omega\mathrm{K} + \frac{2\Omega c_{\alpha,\beta} g^{\alpha\beta}}{c} - \frac{c_{,\alpha\beta}\, b^{\alpha\beta}}{c}. \tag{5}$$

$$\frac{d\bar{r}}{ds} = \bar{\nu}, \quad \frac{d\bar{\nu}}{ds} = -g^{\alpha\beta}\tau_\beta (\ln c)_{,\alpha} \tag{6}$$

where ω is the intensity of the stress jump, Ω, K are the mean and Gaussian curvatures, respectively, $\bar{\nu}$ is the normal to the front, $\bar{\tau}$ is the vector of the tangent to the front. For $g^{\alpha\beta}$, $g_{\alpha\beta}$, $b^{\alpha\beta}$, $b_{\alpha\beta}$, $c^{\alpha\beta}$, $c_{\alpha\beta}$ the equations are also obtained.

For surface waves the system of equations have the form

$$\frac{d\mathrm{X}}{ds} + \frac{1}{4}\frac{d\ln g_{22}^{(s)}}{ds}\mathrm{X} + \frac{1}{2}\frac{d\ln R}{ds}\mathrm{X} = 0, \tag{7}$$

$$\frac{d\ln g_{22}^{(s)}}{ds} = -4\Omega, \quad \frac{d\Omega}{ds} = 2\Omega^2 + \frac{g^{22} c_{,22}}{c}, \tag{8}$$

$$\frac{d\bar{r}}{ds} = \bar{\nu}, \quad \frac{d\nu_i}{ds} = -g^{22} x_{i,2} (\ln c)_{,2} \tag{9}$$

where X is the intensity level of surface waves, g_{22} is the component of a metric tensor.

CONDITIONS OF APPEARING OF THE DETERMINED CHAOS

Investigation of a possibility of appearing of the determined chaos in propagation of waves in heterogeneous media on the basis of the equations (4) - (6) and (7) - (9) to realize practically it is very complicated.

In case of bivariate distribution of a heterogeneity it is possible to split systems of equations (4) - (6) and (7) - (9). Then the equations for rays volume and surface waves manage to be considered separately from other equations. Stochastization of rays causes appearing of chaos in behaviour of dynamic and geometrical parameters at propagation of fronts volume and surface waves in heterogeneous media.

The fluctuations y for rays volume longitudinal and transverse waves about an axis of a waveguide (axis x) are described by the equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}\left[1+\left(\frac{dy}{dx}\right)^2\right]f_1(x,y)+\left[1+\left(\frac{dy}{dx}\right)^2\right]f_2(x,y)=0\,, \tag{10}$$

$$f_1=-c^{-1}\frac{dc}{dx},\quad f_2=-c^{-1}\frac{dc}{dy},\quad c=c_\alpha(x,y),\quad \alpha=l,t$$

where $c(x,y)$ is the wave velocity for longitudinal $\alpha=l$ and for transverse $\alpha=t$ waves.

Expanding functions $f_1(x,y)$, $f_2(x,y)$ in a neighbourhood of an axis of a waveguide at $\left|\frac{dy}{dx}\right|<<1$ we reduce the equation (10) to form

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}f_1-y\frac{\partial f_2(x,0)}{\partial y}-y^2\frac{\partial^2 f_2(x,0)}{\partial y^2}-y^3\frac{\partial^3 f_2(x,0)}{\partial y^3}=\varepsilon F(x,y),\qquad \varepsilon<<1\,. \tag{11}$$

Let's consider some particular cases of distribution of a heterogeneity along an axis of a waveguide. In a case, when along an axis of a waveguide a medium is stratified with piecewise constant properties we obtain the equation

$$\frac{d^2y}{dx^2}+\omega^2(1+\alpha y^2)y=\varepsilon\omega\sum_{k=-\infty}^{\infty}(x-kX)\ \text{where}\ \frac{\partial f_2(x,0)}{\partial y}=-\omega^2,\ \frac{\partial^2 f_2(x,0)}{\partial y^2}\equiv 0,\ \frac{\partial^3 f_2(x,0)}{\partial y^3}=-\alpha\omega^2\,. \tag{12}$$

The going to chaos investigated by behaviour of correlation function $R(r)$ of a phase of a ray. For $R(r)\xrightarrow[r\to\infty]{}0$ happens stochastization of a ray since some x_{st}. The going to stochastic behaviour is stipulated by overlapping of resonances.

Other type of going to chaos is realized for medium in which the heterogeneity along an axis of a waveguide varies periodically. Then the equation is obtained

$$\frac{d^2y}{dx^2}-y+y^3+\varepsilon\delta\frac{dy}{dx}=\varepsilon\gamma\frac{dy}{dx}\cos\omega x\ \text{where}\ \frac{\partial f_2(x,0)}{\partial y}=-1,\ \frac{\partial^2 f_2(x,0)}{\partial y^2}=0,\ \frac{\partial^3 f_2(x,0)}{\partial y^3}=3\,. \tag{13}$$

At the expense of presence in the equation (13) derivative $\frac{dy}{dx}$ the going to a chaotization state is realized through a stochastic attractor.

Analogously the equation for rays in case of surface waves are obtained and investigation. The respective equations are analogous to the equations (12) and (13).

CONCLUSIONS

1. In under waveguide propagation of a ray along an axis of a waveguide an distribution of a heterogeneity is absent the ray oscillates about an axis. At the count of a heterogeneity on this driving in a neighbourhood of a resonant frequency the modulation of a ray on x is superimposed and the amplitude and localization of a ray for a stratified piecewise constant medium is defined.
2. In the second case the existence of stochastic attractors, coherent with a cascade of bifurcations is possible. The trajectory wanders in a neighbourhood of a separatrix until hit yet on an attractor.

Reference

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