OPTIMAL SHAPES OF PARAMETRICALLY EXCITED BEAMS

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Summary Straight elastically supported beams of variable width under the action of a periodic axial force are considered. Two shape optimization problems for reducing parametric resonance zones are studied. In the first problem, the range of resonant frequencies is minimized for a given parametric resonance zone and a fixed amplitude of excitation. In the second problem, the minimal (critical) amplitude of the excitation force is maximized. These two optimization problems are proved to be equivalent in case of small external damping and small excitation force amplitude. It is shown that optimal designs have strong universal character, i.e., they depend only on the natural modes involved in the parametric resonance and boundary conditions. Efficient numerical method of optimization is developed. Optimal beam shapes are found for different boundary conditions and resonant modes. Experiments for uniform and optimal simply supported beams are conducted, which show a very good agreement with theoretical prediction.

INTRODUCTION

Shape optimization of beams under different loading conditions is an interesting and important topic both from theoretical and practical point of view. On one hand, this problem requires sophisticated analytical and numerical methods for sensitivity analysis and knowledge from bifurcation and singularity theories. On the other hand, the correct formulation of the problem, including minimal width constraints and analysis of nonlinear behavior, may strongly influence the practical use of optimal designs. We refer the reader to the surveys on optimal beam problems [1] for the case of static loads and [2] for the case of follower loads. In [3], beams were optimized under several natural frequency constraints. The optimal shape of a pipe conveying fluid was investigated in [4]. Optimal shapes of a beam, minimizing the critical excitation amplitudes of primary parametric resonance zones, were analyzed numerically in [5]. Very few experimental works on optimal beams are available; see [4, 6].

OPTIMIZATION PROBLEMS

In this contribution, we consider beams of rectangular cross-section with constant thickness and variable width. The length and total volume of the beam are fixed. Elastic supports at both ends are considered, which, in particular, include the cases of simply supported and clamped boundary conditions. Parametric resonance of the beam under a periodic axial force is studied. Two optimization problems are considered. In the first problem, the range of resonant excitation frequencies is minimized for a given parametric resonance zone and a fixed amplitude of excitation. In the second problem, the minimal (critical) amplitude of the excitation force is maximized.

UNIVERSAL CHARACTER OF OPTIMAL DESIGNS

We prove that the two optimization problems under consideration are equivalent in case of small external damping and a small excitation force amplitude. Moreover, we show that the optimal designs do not depend on the damping coefficient, on the value of the excitation amplitude in the first problem, and on the resonance number. All these properties reveal strong universal character of optimal beam shapes, which depend only on the natural modes involved in parametric resonance and on the boundary conditions. This universality is of great importance for practical use of optimal designs. Numerical computations show that the optimal shape changes weakly if the parametric excitation is applied to a prestressed beam (with a static axial force lower than the critical Euler force).

NUMERICAL METHOD AND RESULTS

In the analysis, we use explicit formulae obtained recently in [7] (see also [8]), which describe the parametric resonance zones as half-cones in the three-parameter space (frequency and amplitude of parametric excitation and damping coefficient). As a result, the optimization problem is reduced to the minimization of an objective functional depending only on natural frequencies and modes involved in parametric resonance. This approach allows avoiding time-consuming multiple integrations of equations of motion required for stability analysis in the Floquet theory, see [8]. Optimal beam shapes are found numerically for different boundary conditions and resonant modes.

EXPERIMENTS

Experiments on simple parametric resonance of the first mode are conducted for the uniform and optimal simply supported beams. The reduction of the resonance zone for the optimal beam is shown to be in very good agreement with theoretical prediction. Nonlinear response of the uniform and optimal beams is analyzed experimentally.

COMPARISON WITH PREVIOUS WORKS

Compared to the previous work [5], where the beam of circular cross-section was optimized, the present study has the following advantages. First, we use a more realistic model of external damping taking into account the beam width variation. We consider two different optimization problems and prove universality properties of our optimal designs. Finally, the gain in optimization attained in our study is about 17–22% (depending on the boundary conditions and width constraints), which is higher than 4–9% obtained in [5].

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