THE PLANE CRACK IN A FUNCTIONALLY GRADED ORTHOTROPIC STRIP

Linzhi Wu, Licheng Guo and Li Ma

(*Center for Composite Materials, Harbin Institute of Technology, Harbin 150001, P.R. China*)

Summary The plane crack problem of a functionally graded orthotropic strip under concentrated loads is studied. The edge crack is perpendicular to the boundary and the elastic property of materials is assumed to vary continuously along the thickness direction. By using integral transform method, the present problem can be reduced to a singular integral equation. The influences of geometric and material's parameters on stress intensity factors are analyzed.

FORMULATION OF PROBLEM

Though there are a lot of papers related to the crack problem of FGMs, very few papers on the plane crack problem of a functionally graded strip with a crack perpendicular to the boundary are published. It is very significant to investigate this kind of crack problems since the model can be used as an approximation to a number of structural components and laboratory specimens.

Consider the plane crack problem of a functionally graded orthotropic strip with properties varying along the x-axis (see Fig.1). The strip is infinite along the y-axis and has a thickness *h* along the x-axis. The edge crack is perpendicular to the boundaries. The material properties are defined as

$$c_{11}(x)=c_{110}e^{\delta x}, \quad c_{12}(x)=c_{120}e^{\delta x}, \quad c_{22}(x)=c_{220}e^{\delta x}, \quad c_{66}(x)=c_{660}e^{\delta x} \tag{1}$$

where $c_{110}, c_{120}, c_{220}, c_{660}$ and δ are constants.

Substituting the constitutive equation into the equilibrium equation and using Fourier's transform technique, we can obtain the corresponding displacement field and stress field. Introducing the auxiliary function and according to boundary conditions, after some manipulation, we can obtain the following integral equation for the edge crack

$$\int_0^b [h_1(u,x)+h_2(u,x)]g(u)du = -e^{-\delta x}[\sigma(x)+\frac{2}{\pi}\int_0^{\infty} X_3 X_1^{-1} X_2 ds], \quad 0 \le x < b \tag{2}$$

where the right hand of equation (2) is the known function, and

$$h_1(u,x)=\lim_{y\to 0}\frac{1}{2\pi}\int_{-\infty}^{+\infty} K_1(y,s)e^{is(x-u)}ds, \quad h_2(u,x)=\lim_{y\to 0}\frac{2}{\pi}\int_0^{+\infty} K_2(u,x,s)\cos(sy)ds \tag{3}$$

To derive the singular integral equation, an asymptotic analysis is needed. When $y \to 0$ and $s \to \infty$, the asymptotic form of $K_1(y,s)$ can be expressed as

$$K_{1\infty}(0,s)=w_{11}+w_{12}/s \tag{4}$$

where w_{11} and w_{12} are known constants related to the material constants.

RESULTS AND DISCUSSIONS

Let's consider the edge crack of a functionally graded orthotropic strip as shown in Fig.1. In numerical calculations, two kinds of orthotropic materials given by Chen et al.[1] are chosen. The constants C_{110}, C_{120}, C_{220} and C_{660} are taken as 1.048×10^{10}, 3.248×10^{9}, 1.578×10^{10} and 7.07×10^{9}, respectively.

Fig.2 shows the variations of normalized stress intensity factors with the crack length $a_0 = b/2h$ when the nonhomogeneity constant δh takes different values. It can be found that $k_1(b)/k_0$ increases with the increasing of the crack length. However, $k_1(b)/k_0$ decreases with the increasing of the nonhomogeneity constant δh. Therefore, the normalized SIFs are greater when the edge crack lies in the stiffer side of the strip. To investigate the influence of loading conditions on SIFs, we take $r_0 = 3.0$. From Fig.3, it is found that normalized SIFs increase with the increasing of $a_0 = b/h$ for the same nonhomogeneity constant δh. By further observation, it can be found that $k_1(b)/k_0$ increases with the increasing of δh. Here, $\delta h < 0$ implies that the crack side is stiffer than the other side, while $\delta h > 0$ means the crack side is softer than the other side of the strip. Therefore, it can be concluded that the normalized SIFs are greater when the edge crack lies in the stiffer side of the strip. By comparing Fig.2 with Fig.3, we see that the applied loads have important effects on the distribution of stress intensity factors near the crack tip. Similarly, the present works can be expanded to investigate the internal crack problem of FGMs.

CONCLUSIONS

For the edge crack problem, to our knowledge, it is the first time to provide the asymptotic expression in the singular integral equation. The stress intensity factors for the mode I problem are solved under two different loading conditions, namely crack surface pressure and fixed-grip loading. The effects of material constants and geometry parameters on stress intensity factors are studied. It is found that the effect of the material nonhomogeneity constant on stress intensity factors is very significant. For the uniform crack surface pressure, normalized SIFs near the stiffer crack tip of the strip are greater. For the fixed-grip loading, normalized SIFs near crack tips increase with the increasing of δh.

References

[1] Chen J, Liu Z.X., Zou Z.Z. Transient internal crack problem for a nonhomogeneous orthotropic strip (Mode I). Int J Eng Sci, 40: 1761-1774, 2002.

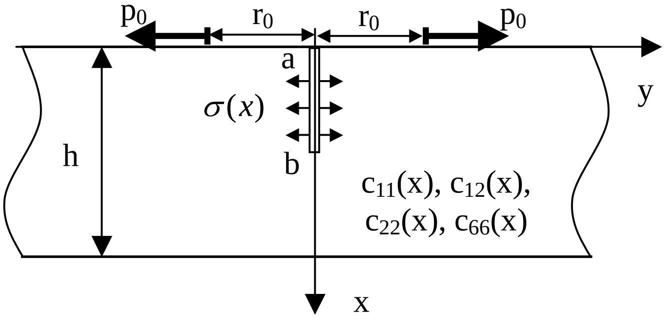

Fig.1 The schematic of the orthotropic functionally graded strip with an edge crack

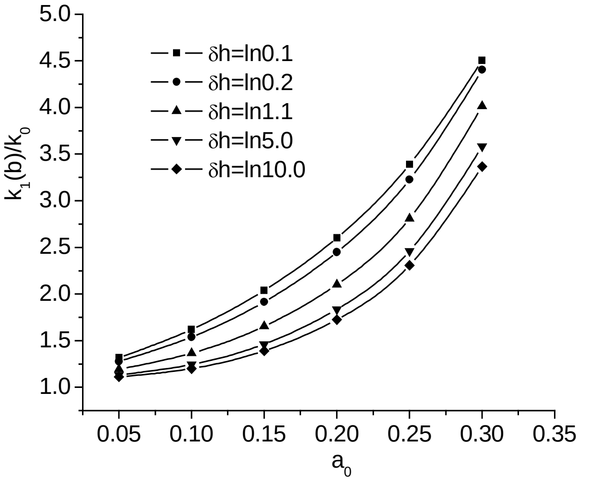

Fig.2 Normalized SIFs of the edge crack under uniform crack surface pressure

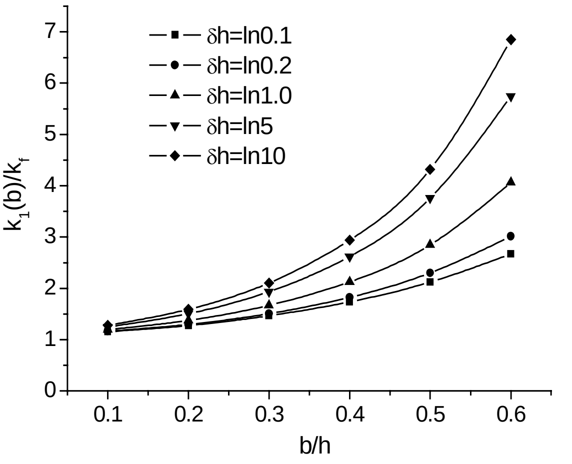

Fig.3 Normalized SIFs of the edge crack under the fixed-grip loading