

FRICIONAL SLIP BETWEEN A GRADIENT NON-HOMOGENEOUS LAYER
AND A HALF-SPACE IN ANTI-PLANE ELASTIC WAVE FIELD

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EXTENDED SUMMARY

The problem to treat here is shown As in Fig.1. An elastic layer with thickness H is pressed a elastic half-space by an applied pressure p^o . Both the layer and half-space are composed of the gradient non-homogeneous medium, and suppose that there is the Coulomb friction with static and kinetic friction coefficients f_s and f_k at the interface between them The anti-plane motions in the layer and half-space will be aroused under the action of SH parabolic pulse incidence wave, as shown in Fig.2. When the shear elastic modulus $m(m^*)$ and the density $r(r^*)$ of medium vary by the laws of the second power in the gradient direction of medium, the analytical solution of the problem is found as

$$w = \int_{-\infty}^{+\infty} [A(k)(1+ay)^{\frac{1}{2}} H_{\frac{1}{2}}^{(1)}(p \frac{k}{a}(1+ay)) + B(k)(1+ay)^{-\frac{1}{2}} H_{\frac{1}{2}}^{(2)}(p \frac{k}{a}(1+ay))] e^{ikx} dk \tag{1}$$

$$w^* = \int_{-\infty}^{+\infty} [A^*(k)(1+a^*y)^{\frac{1}{2}} H_{\frac{1}{2}}^{(1)}(p^* \frac{k}{a^*}(1+a^*y)) + B^*(k)(1+a^*y)^{-\frac{1}{2}} H_{\frac{1}{2}}^{(2)}(p^* \frac{k}{a^*}(1+a^*y))] e^{ikx} dk \tag{2}$$

$w(w^*)$ is the anti-plane displacement; a, a^* are two known gradient constants of

media; $p = \sqrt{\left(\frac{c}{c_s(c_s^*)}\right)^2 - 1}$; c is the apparent velocity of elastic wave along the

interface $c_s(c_s^*)$ is shear wave speed of medium; A, B, A*, B* are known

functions of k; asterisk * refers to the quantities of the layer; $H_{\frac{1}{2}}^{(1)}, H_{\frac{1}{2}}^{(2)}$ are Hankel

function of 1,2 kind of 1/2 order. From these, he distribution of dimensionless shearing traction

$\mathbf{t}(\mathbf{h})$ and that of dimensionless relative slip $\mathbf{d}(\mathbf{h})$ are obtained as

$$\begin{aligned} \mathbf{t}(\mathbf{h}) = \operatorname{Re} \int_{-\infty}^{+\infty} & \left(H_{\frac{3}{2}}^{(2)}\left(p \frac{k}{\mathbf{a}}\right) + \frac{A(k)}{A^i(k)} H_{\frac{3}{2}}^{(1)}\left(p \frac{k}{\mathbf{a}}\right) \right) \\ & \times \frac{2p\left(\frac{h}{l}\right)}{pkH_{\frac{1}{2}}^{(2)}\left(p \frac{k}{\mathbf{a}}\right)} \left(\frac{\sin kl}{kl} - \cos kl \right) e^{ikx} dk \end{aligned} \quad (3)$$

$$\begin{aligned} \mathbf{d}(\mathbf{h}) = \operatorname{Re} \int_{-\infty}^{+\infty} & \left(H_{\frac{1}{2}}^{(2)}\left(p \frac{k}{\mathbf{a}}\right) + \frac{A(k)}{A^i(k)} H_{\frac{1}{2}}^{(1)}\left(p \frac{k}{\mathbf{a}}\right) \right) \\ & + \left[\frac{A^*(k)}{A^i(k)} H_{\frac{1}{2}}^{(1)}\left(p^* \frac{k}{\mathbf{a}^*}\right) + \frac{B^*(k)}{A^i(k)} H_{\frac{1}{2}}^{(2)}\left(p^* \frac{k}{\mathbf{a}^*}\right) \right] \\ & \times \frac{2\left(\frac{h}{l}\right)}{pkH_{\frac{1}{2}}^{(2)}\left(p \frac{k}{\mathbf{a}}\right)} \left(\frac{\sin kl}{kl} - \cos kl \right) e^{ikx} dk \end{aligned} \quad (4)$$

Take $\mathbf{a}l = \mathbf{a}^*l = 0.2$; $f_0 = f_k / f_s = 0.8$; $\mathbf{m}^* / \mathbf{m} = 1.5$; $ph/2l = 1$; $c^* / c = 1.2$;

$\mathbf{q}_0 = \sin^{-1}(c_s / c) = \mathbf{p} / 6$; $H/l = 50$; $\mathbf{x} = (x - ct)/l$. $A^i(k)$ incident wave amplitude;

Fig.3 shows the variation of $\mathbf{t}(\mathbf{h})$ with \mathbf{h} (real line) and the extent and location of the stick

and slip zone for given parameters. It can be seen from the figure that the state of the interface reveals periodicity. The periodic length is 50. There are 3 slip zones (2 in right direction and 1 in inverse direction) in a periodic length, and on which put a number (1), (2) or (3), respectively. As the dimensionless external pressure \mathbf{s} increase, the number of the slip zone will increase, its

extent become larger. Fig.4 shows the distribution of the dimensionless shear traction $\mathbf{t}(\mathbf{h})$ and

dimensionless relative slip $\mathbf{d}(\mathbf{h})$ in the slip zone (1)-(3) and neighborhoods of them. It can be

seen from the figures that the shear traction $\mathbf{t}(\mathbf{h})$ at the leading edge of the slip zone has a jump,

but is continuous at the training edge, which is caused by the difference between the static and kinetic friction. Besides, there is a rigid displacement between the leading edge and the training edge. In order to reveal the effect of the interface thickness, we calculate the variations of the

dimensionless shear traction \mathbf{t} (33) and the dimensionless relative slip \mathbf{d} (33) with the

dimensionless layer thickness H/l , as shown in Fig.5. From the figures we see that the change the dimensionless layer thickness H/l will make the state of the interface change, but the change is not monotonous.

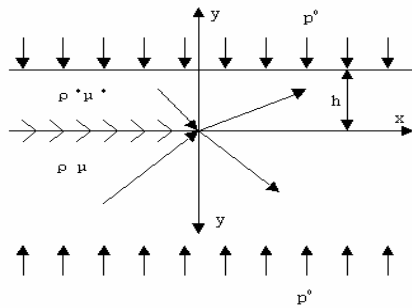


Fig.1 Anti-plane wave field in a half-space with a layer

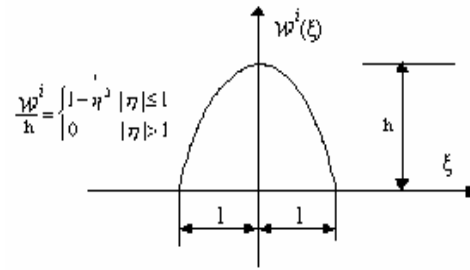


Fig.2 Incident parabolic pulse

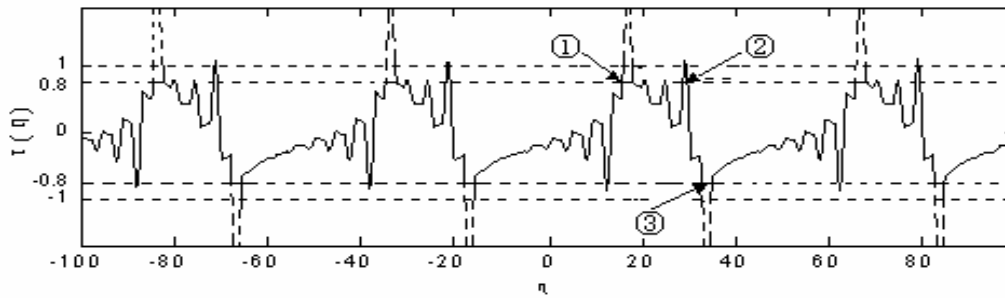


Fig.3 The stick and slip zones and distribution of shearing traction $\tau(\eta)$ at the interface

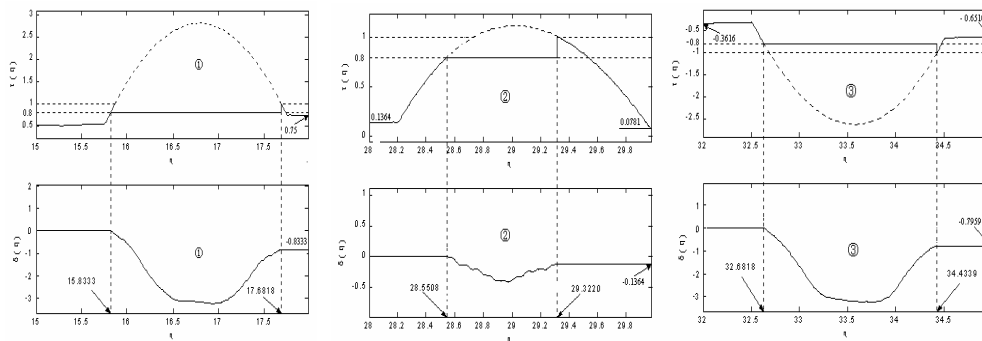


Fig. 4 Distribution of shearing tractions $t(h)$ and relative slip $d(h)$ at point and its neighborhood

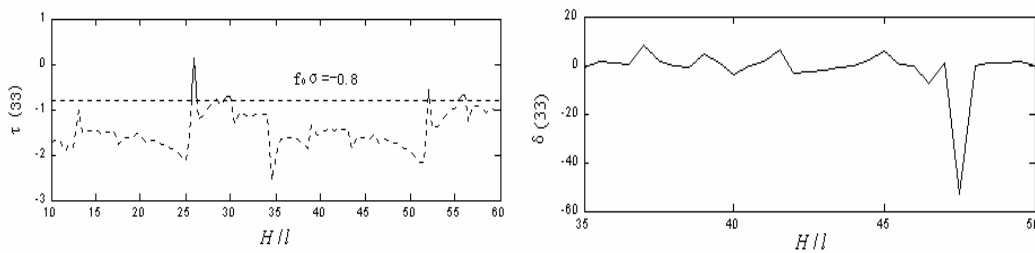


Fig.5 Variations of shearing traction $t(33)$ and relative slip $d(33)$ with dimensionless layer thickness H/l (real linear)

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