

A NUMERICAL APPROACH FOR LARGE-SCALE COMPUTATION CEM

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Summary A new numerical approach for large-scale computation-cellular element method (CEM) is introduced. The similarities and differences among the moment distributed method, finite element method and cellular element method are studied. Its basic principle and computing steps are presented. Several numerical computing examples are given. Based on the numerical results, the feasibility, advantages and disadvantages of cellular element method are discussed.

THE BASIC PRINCIPLE OF CELLULAR ELEMENT METHOD

At setting up the material constitutive relation, there is principle of local action. It isn't necessary to consider the motion of particles outside its vicinal area when you decide the stress of a particle. That is to say, the action of a particle is regarded as being limited in a definite area. Its motion and stress only decide on the motion of its adjacent particles because it is numb to the information of other particles. Material equilibrium is considered an automatic organic phenomenon through the information transmission of each other. The course is very similar to cellular automata (CA).

A cellular automata is a mathematical model dispersed in time and space. According to it, any system consists of disjunctive individuals. It can be used for describing the whole action behaviour and the evolution process of any system displayed by particles' interaction. CA usually describes cell position in scalar at present, and evolves according to a certain rule, finally attains a kind of stable state. We can generalize the basic characteristics of the cellular automatic as follows: (a) A lot of dispersed meshes and nodes called cells constitute a system (one, two and three dimension). (b) All cells' positions are described by discrete parameters or continuous parameters. (c) All cells are only connected with their adjacent area in definite space, and the current position of a cell is decided on the area of its neighbour. (d) The state of a cell will change with a certain regulation (the characteristic of cell).

Therefore, the analysis of the mechanics fits the request of cellular automata's four basic characteristics, and the characteristic (d) is the force-deformation curve that is decided by material constitutive relation. Certainly, the state of each cell must be described by vector or tensor. Because the number of particles in a structure is infinite, it is very difficult to directly study each particle's mechanics behaviour and evolution mechanism. But we can study finite particles instead of infinite particles. It is the same to finite element method in which a structure is regarded as a composite that consists of finite cells. We can attain the approximate solution of all elements or nodes through the way. Previously we must analysis the cells mechanics characteristic by dividing elements and using the discrete techniques on condition that the static equilibrium equations are obeyed.

For a structure, we can disperse it as in finite element method and get cells formed with nodes and elements. A node and its vicinal nodes and elements form a cell. The cell that node N_i is on its center is called cell D_i . Supposing N_i has relation with E_i^j formed by number of m_i elements. Here N_i^k are their adjacent nodes. Then it is describe as

$$D_i = N_i \cup \{E_i^1, \dots, E_i^{m_i}; N_i^1, \dots, N_i^{n_i} \mid N_i \in E_i^j; N_i^k \in E_i^j, j=1, \dots, m_i\} \quad (1)$$

The cell D_i includes vicinal nodes N_i^k ($k=1, \dots, n$) beside N_i and correlative elements E_i^j ($j=1, \dots, m_i$). The state V_i of cell D_i is defined as general displacements (u_i) and forces (F_i). Then there is equation:

$$V_i = \{u_i, F_i\} \quad (2)$$

V_i has $2R$ dimension. It may be $R=2$ (plane truss, elastic plane problem and axial symmetry structure), $R=3$ (plane frame, slab, three-dimension object) or $R=6$ (space frame and shell). It will change with various structures.

For cell D_i , the forces on node (F_i) and the restrictions of its neighbour node N_i^k will decide the displacements of node N_i . The relation of forces and deformations can be analysed by two proceedings with the form of its increment:

(1) Nodal force increments ΔF_i will result in displacement increments Δu_i of the node N_i .

(2) With the displacements increments Δu_i , the node N_i may cause its adjacent nodes to have the nodal force increments ΔF_i^k ($k=1, 2, \dots, n_i$).

All cells of the structure transit force-displacement-force each other. Finally the increments of Δu_i and ΔF_i^k will be nearly equal to zero. This automatic adjusting phenomenon will make the structure equilibrium.

Based on principle of local action, the idea of Cellular Automata and the discrete techniques in finite element method, the new numerical analysis method is called cellular element method (CEM).

THE ALGORITHM OF CELLULAR ELEMENT METHOD

The general solving process of cellular element method can be described as follows:

- (1) Divide the structure into a series of elements and nodes by using the discrete techniques in finite element method.
 - (2) For each node, circular operation as follows is done.
 - (a) Applying zero-displacement restrictions on the adjacent nodes of the computing node, i.e., make them fixed.
 - (b) Calculating the displacements of the computing node. The region where the computing node and its adjacent nodes can be seen as a cell, and the restricting condition is that its adjacent nodes are fixed. Only one node (the current computing one) can move. Its displacements can be solved with finite element method (rigidity method). We only have to solve a equation group with 6 linear equations at most, because the number of a node's displacements is not more than six.
 - (c) Calculating the restriction forces of the adjacent nodes.
- We can calculate the nodal forces of the element by using the computing node's displacements and the zero-displacement of its adjacent nodes. In the same time, the restriction forces of the adjacent nodes can be worked out.
- (d) Assuming the loads of the computing node are zero, and the restriction forces of the adjacent nodes are added to the nodes negatively.
 - (e) Accumulating the displacements of the computing node.
- (3) At the end of each circulation, check whether the differences (named ε) of every node's displacements in two circulations are in the range of accuracy. If not, we continue the next circulation. Finally, we will attain the approximate value of every node's displacements in balance state.
 - (4) Computing the stresses and strains of elements with the displacements of nodes, elements' geometric and physical equations.

The method has the following advantages:

- (1) It isn't necessary to solve the structural whole balance equation, and has not problems caused by a large quantity of data memory when computing on computer.
- (2) It has complete parallel computing ability.

The solving process of cellular element method is similar to the moment distributed method in structure mechanics. In fact, the moment distributed method can be seen as a particular example of cellular element method in solving a rigid frame structure.

But in the classical moment distributed method, each node had only one direction displacement. While moment distributed method can only be suit for continuous beam and plane rigid frame without lateral displacement, cellular element method can be applied to any structure.

NUMERICAL INSTANCES

Three numerical instances were given: a space frame structure, a plane stress problem, and a space entity structure. All results show that when we use the cellular element method to calculate the structures, we can get the same solutions as those worked out by finite element method.

CONCLUSIONS

Cellular element method is a new numerical computing method. In this method the integer analysis of structure is changed into a series of local element analysis by using the idea of Cellular Automata. It may have a good future in the aspect of computing large structure because it hardly has any special request on the capability of computer. Simultaneity, It can be developed into a high parallel arithmetic to suit for the request of parallel computer.

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