INTRUSIVE GRAVITY CURRENTS IN A STRATIFIED AMBIENT - NOVEL THEORETICAL RESULTS AND INSIGHTS

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Summary The intrusion of a fixed volume of "mixed" fluid which is released from a lock (of rectangular or cylindrical shape) and then propagates horizontally at the neutral buoyancy level in a stratified ambient fluid is considered. The investigation, based on a closed one-layer shallow-water model supported by comparisons with experiments and Navier-Stokes numerical results, provides new and accurate insights for the initial motion.

We consider the mainly horizontal motion which develops when a fixed volume of fluid of constant density, ρ_c, is released from a lock of length x_0 and height h_0 into a vertically stratified ambient at the level of neutral buoyancy, see Fig. 1. We assume that the density of the ambient fluid, $\rho_a(z)$, varies linearly over a layer of finite thickness, l, or over the full depth of the container, H, and that the Reynolds number is large. The problem is relevant to natural "mixed region collapse" in oceans and atmosphere, hydraulic reservoirs management, spread of pollutants and sewage, etc. The previous investigations were concerned with both experimental observation (mainly with salt-water systems) and theoretical interpretations ([1]-[8], [10]). However, important gaps remained open in the theoretical understanding and formulation, including some confusion about the scaling of the variables and the effect of the shape of the lock. Wu [10] suggested a curve-fit description for the position of the nose x_N as a function of time t, based on a very restricted set of data acquired in a configuration with a cylindrical lock. Kao [3] and Manins [4] developed box-model approximations for $x_N(t)$ under the assumption that intruding fluid has a fixed shape (an ellipse or a rectangle); both models used adjustable parameters based on some ad-hoc matchings. Actually, no predictive model based on governing balance equations for the initial stages of the lock-release and related problems has been presented, and the "state-of-the-art" was accurately summarized by Faust and Plate [2]: "intrusions into a linearly stratified environment behave very differently from theoretical calculations". The need to close this gap provided the motivation of the work reported here.

The present work introduces a new analysis, based on the inviscid Boussinesq-fluid shallow-water (SW) equations of motion and backed by numerical solutions of the Navier-Stokes (NS) problem. This SW formulation provides a hyperbolic system for the (z-averaged) velocity u and (half-) thickness h of the intruding fluid as functions of t and x, subject to realistic initial and boundary conditions, and uses no adjustable constants. From this model many essential features of the motion are derived: the constant velocity in the initial slumping stage (for rectangular lock configuration), the propagation with $t^{1/2}$ at large times, the dependency of the velocity on the thickness of the density transition layer l, the differences between the flows generated by rectangular and cylindrical locks, the "sub-critical" property of the velocity of propagation (as compared with the mode 2 linear waves). The analytical results were corroborated by comparisons with experiments and finite-difference simulations based on the NS equations.

The analytical model is an extension of the one-layer shallow-water formulation developed by Ungarish and Huppert [9] for the investigation of gravity currents which propagate at the bottom of a linearly-stratified ambient. A major simplification is the assumption that the ambient can be considered quiescent for some significant initial period of time. We show that the neglect of the internal gravity waves in the SW formulation is justified for some initial period of propagation, and we estimate the position where the nose-waves interaction becomes important. We show that for a deep-ambient configuration the SW formulation admits a similarity solution with $x_N(t) = Kt^{1/2}$, where K is a constant that reflects the volume of the mixed fluid (note the difference from the $t^{2/3}$ propagation of the classical gravity current in a homogeneous ambient). We also show that the previous analytical models of Kao and Manins are sub-sets of the present formulation.

The theory predicts that an intrusion released from a rectangular lock will propagate during the initial slumping stage with constant velocity, u_N, like a classical gravity current. In contrast, the propagation after release from a cylindrical lock displays a clear acceleration-deceleration pattern; the maximum velocity is, remarkably, equal to the slumping u_N of a counterpart rectangular lock configuration.

Comparisons with experimental data ([1], [2], [7], [10]) were performed, and consistency with theory was obtained. The discrepancy between the experimental and analytical velocities of propagation is typically about $2-10\%$. The differences are not systematic and can be attributed to experimental errors and deviations from the idealizations of the models. Overall, we think that the agreement is very good, actually as good as can be expected from the SW theory (in the classical homogeneous circumstances similar errors were obtained).

Typical results of propagation, x_N as a function of t, are shown in Fig. 2. Again, agreement between the SW solution and the experiment is very good (the discrepancies can be attributed to viscous and mixing effects). We emphasize that the theory has been developed from first principles without the use of any adjustable parameters or other information taken from the experiment. For the rectangular lock configuration in Fig. 2 at $t \approx 6$ a strong deceleration of the intrusion is indicated by both experiment and NS simulation. This is attributed to the interaction between the nose and the internal gravity waves, an effect not resolved by the SW formulation. An estimate for the position where this interaction occurs has been derived, in good agreement with observations; afterwards, the SW formulation needs modifications. Directions of further research will be discussed.

References

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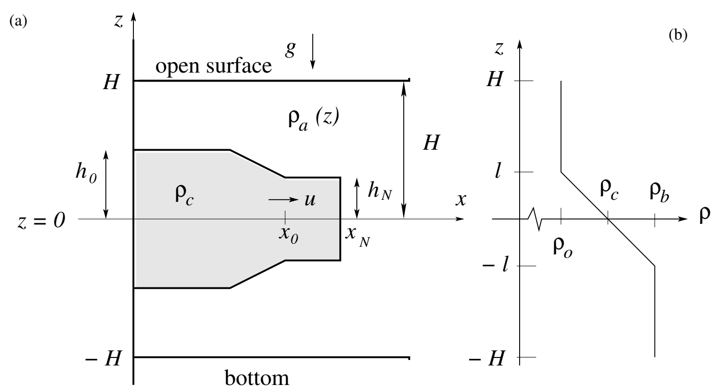

Figure 1: Schematic description of the system: (a) geometry after release from a rectangular lock; (b) density profile in the ambient (note $\rho_c = 0.5(\rho_b + \rho_o)$). In dimensionless form, vertical and horizontal lengths are scaled with x_0 and h_0, respectively. Velocity and time are scaled with $U = (g'h_0)^{1/2}$ and x_0/U, where $g' = (\rho_c/\rho_o - 1)g$. The subscripts denote: N - nose (or front); a - ambient; b - bottom; c - current (intrusion); o - open surface.

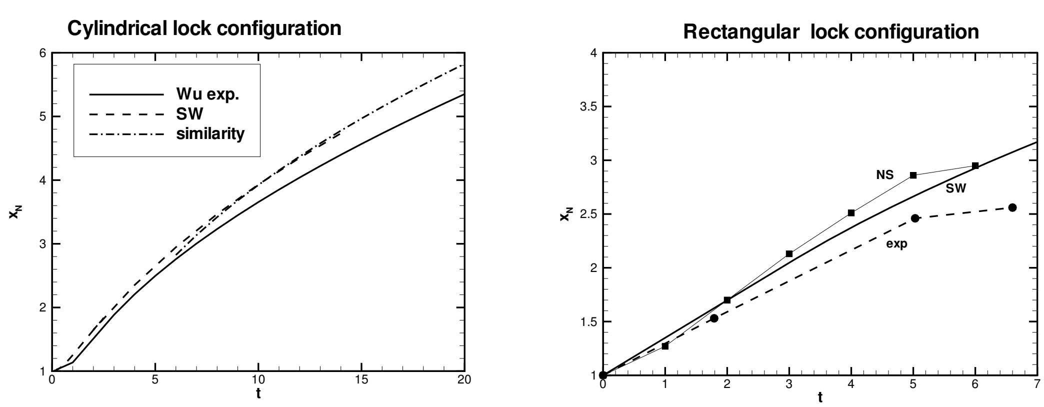

Figure 2: Propagation results x_N as a function of time, in full-depth linearly stratified ambient ($l = H$), dimensionless. The cylindrical lock configuration corresponds to the experiments of Wu [10], and comparison is with the present SW results and similarity solution. The rectangular lock configuration is for experiment run 117 of Amen and Maxworthy [1], and comparison is with the present SW and Navier-Stokes results.