FRACTURE CRITERIONS FOR BRIDGED CRACK: FROM MACRO TO NANOSCALE

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Summary A multi-scale bridged crack model for the evaluation of fracture and strength parameters of interfaces with cracks is proposed. We suppose that distributed nonlinear bonds link the crack surfaces in some zones starting from the crack tips. The sizes of these zones are not assumed to be small as compared to the crack length on every bridging level. The system of singular integral-differential equations is derived for normal and shear bond stresses evaluation. Two fracture criterions of quasistatic crack growth are considered. The first condition in the both criterions is the same: it is the condition of the bond limit stretching or the limit strain at the trailing edge of the bridged zone. Two types of the second condition of fracture is considered: a) the force condition which is the condition for the critical stress intensity factor and b) the proposed energetic condition based on the equality of the values of the strain energy release rate and the energy dissipation by the bonds at the crack limit equilibrium state. The regimes of the bridged zone and the crack tip equilibrium and growth are analyzed for the both types of these criterions.

INTRODUCTION

Let us consider a straight crack of length 2ℓ at an interface of two dissimilar elastic half-planes such that the crack is placed at $|x| \leq \ell, y = 0$. Assume that the uniform tensile stresses, σ_o, are applied at infinity normal to the interface. Consider segments of length d (end zones) adjacent to the tips of the crack $(\ell - d) \leq |x| \leq \ell$. In these zones the surfaces of the crack interact with each other, which suppresses the crack opening. The physical nature of the crack surfaces interaction is generally changed depending on the crack scale and distance from the crack tip. The interatomic and intermolecular forces, bundles of nanotubes are limiting mechanisms of the surfaces interaction for a crack of small size or at the very small distances from the crack tips for large crack, while "macro-mechanical" forces prevail at relatively larger distances. These macro-mechanical forces can be caused, e.g., by reinforcing action of fibers in composites or polymer chains connecting the crack surfaces in polymer-polymer joints or polymer joints with other materials (metals, ceramics and etc). To mathematically describe the interaction between the surfaces of the crack, we assume that there exist bonds between the surfaces of the crack at the end zone. The law of deformation of these bonds, which is generally nonlinear, is given. Under the action of external loads, σ_0, the stresses $Q(x)$ appear in the bonds between the surface of the interface crack at the boundary between different materials. These stresses have the normal $q_y(x)$ and tangential $q_x(x)$ components $Q(x) = q_y(x) - iq_x(x),\ i^2 = -1$. The surfaces of the crack are acted on by the normal and tangential stresses which are numerically equal to these components. The opening of the interface crack, $u(x)$ at $|x| \leq \ell, y = 0$, can be written as follows $u(x) = u_y(x) - iu_x(x)$ where $u_y(x) = u_y^+(x) - u_y^-(x)$ and $u_x(x) = u_x^+(x) - u_x^-(x)$ are the projections of the crack opening on the coordinate axes, u_x^+, u_y^+ and u_x^-, u_y^- denote the components of the displacements of the upper and lower crack surfaces. The relation between the crack opening and the bond tractions (the bond deformation law) depends on the physical origin of the bonds and their properties. In the case of spring-like bonds the deformation law can be written as follows [1] $u_i(x) = c_0(x,\sigma)q_i(x)$, $c_0(x,\sigma) = \gamma_0(x,\sigma)\frac{H}{E_B}$, where γ_0 is a dimensionless function, H is a linear scale proportional to the bonding zone thickness, E_B is the effective Young modulus of the bonds and the function c_0 can be considered as the effective bond compliance, $\sigma = \sqrt{q_x^2 + q_y^2}$ is the modulus of the traction vector. For the bond deformation law with the displacements depends on the bonds stresses the crack opening along the crack end zone and the bond stresses can be determined from the solution of the singular integral-differential equations system, the detail can be found in [1].

TWO PARAMETRIC FRACTURE CRITERION

Supposing that the bonds stresses and the crack opening along the crack end zone are known, the total potential energy of a body containing a crack with bridged zone (in the absence of body forces) is

$$\Pi = \int\limits_v w(\varepsilon_{ij})dv - \int\limits_{s_e} t_i u_i ds + \int\limits_{s_i} \Phi(u)ds, \tag{1}$$

where $w(\varepsilon_{ij})$ is the density of the deformation energy in the body volume v, ε_{ij} are the components of the strain tensor; t_i, u_i are the tractions and displacements at the body boundary and (or) crack surfaces s_e; $\Phi(u)$ is the density of the strain energy of the bonds in the crack end zones, u is the crack opening in the end zones of area s_i. The crack limit equilibrium corresponds to the following condition

$$-\frac{\partial \Pi}{\partial \ell} = -\frac{\partial}{\partial \ell}\left[\int\limits_v w(\varepsilon_{ij})dv - \int\limits_{s_e} t_i u_i ds\right] - \frac{\partial}{\partial \ell}\int\limits_{s_i} \Phi(u)ds = 0 \tag{2}$$

The terms in the brackets represent the strain energy release rate at creation of a new crack surface and the last term is the rate of the energy absorption in the crack end zone and is associated with the energy necessary to create a unit of its new surface. Note, that within the framework of the model the rate of the energy absorption depends on the end zone size and bond characteristics. The equilibrium end zone size is not assumed to be constant. The strain energy release rate in the case of an interface crack under the external loading σ_0 and the stresses $-Q(x)$ applied to the crack surfaces in the bridged zone can be written as follows [3] $G_{tip}(d,\ell) = \left(\frac{k_1+1}{\mu_1} + \frac{k_2+1}{\mu_2}\right) \frac{K_B^2}{16\,\cosh^2(\pi\beta)}$, where $k_{1,2} = 3 - 4\nu_{1,2}$ or $k_{1,2} = (3-\nu_{1,2})/(1+\nu_{1,2})$ for the plane strain or plane stress states, respectively; $\nu_{1,2}$, $\mu_{1,2}$ are the Poisson ratios and shear moduli of the joint materials $1(y>0)$ and $2(y<0)$, $\alpha = (\mu_2 k_1 + \mu_1)/(\mu_1 k_2 + \mu_2)$, $\beta = \ln\alpha/2\pi$ and $K_B = \sqrt{K_{\rm I}^2 + K_{\rm II}^2}$ is the modulus of the stress intensity factors due to the external loads and the stresses in the crack end zone. The stress intensity factors $K_{I,II}$ are determined by [1, 2] .

Let us calculate the rate of the energy absorption for the interface crack with bonding. Denote by $U_{bond}(d,\ell)$ the work of the deformation of bonds and by $G_{bond}(d,\ell)$ the rate of the energy absorption per unit thickness of the body. Then

$$U_{bond}(d,\ell) = b\int\limits_{\ell-d}^{\ell} \Phi(u)dx\ , \quad G_{bond}(d,\ell) = \frac{\partial U_{bond}(d,\ell)}{b\ \partial\ell} \tag{3}$$

where b is the body thickness. The density of the strain energy of the bonds is equal to $\Phi(u) = \int\limits_0^{u(x)} \sigma(u)du$, $\quad u(x) = \sqrt{u_x^2(x) + u_y^2(x)},\quad \sigma = \sqrt{q_x^2 + q_y^2}$. After differentiation in formula (3) with respect to the upper and the bottom limits of the integral we can get

$$\frac{\partial U_{bond}(d,\ell)}{b\partial\ell} = \int\limits_{\ell-d}^{\ell} \left(\frac{\partial\, u(x)}{\partial\,\ell}\sigma(u)\right) dx + G_c - G_b, where, G_c = \int\limits_0^{u(\ell)} \sigma(u)du; \quad G_b = \int\limits_0^{u(\ell-d)} \sigma(u)du \tag{4}$$

If we consider the model of the crack with zero opening at the crack tip $(u(\ell) = 0)$ then $G_c = 0$ and it is necessary to add in the left part of (4) the value of the intrinsic toughness of the matrix material $G_c = 2c_m\gamma_m$, where c_m is the volume fraction of the matrix material and $2\gamma_m$ is the matrix toughness. Finally, we obtain the following expression for the rate of the energy absorption

$$G_{bond}(d,\ell) = \int\limits_{\ell-d}^{\ell} \left(\frac{\partial\, u_y(x)}{\partial\,\ell} q_y(u) + \frac{\partial\, u_x(x)}{\partial\,\ell} q_x(u)\right) dx - \int\limits_0^{u(\ell-d)} \sigma(u)du + G_c \tag{5}$$

where the second term is the density of deformation energy allocated at break of the bond at the trailing edge of the crack end zone.

For a homogeneous material or an adhesion layer connecting different materials the following relations are held $G_c = G_b = \int\limits_0^{u(\ell-d)} \sigma(u)du$. In this case the expression (3) completely coincides with similar expression from [1, 2].

The condition of the crack tip limit equilibrium can be written as follows

$$G_{tip}(d,\ell) = G_{bond}(d,\ell) \tag{6}$$

The condition (6) is necessary but insufficient for searching for a limit equilibrium state of the crack tip and the end zone. To search for the limit state of both the crack tip and end zone one should introduce the condition of the bond limit stretching at the trailing edge of the end zone. Solving jointly these critical equations we can determine the critical external loads, the end zone size d_{cr}, and the adhesion fracture resistance $G_{tip}(d_{cr},\ell)$ at the crack limit equilibrium state for the given crack length and the bond characteristics. In this part of the abstract the energetic fracture criterion is considered in the detail. The description of the force fracture criterion can be found in [2].

CONCLUSIONS

The application of the proposed model and the comparison the criterions for different problems from macro to nanoscale of the interface cracks are presented. It is noted that the most differences between the results from the force and energetic fracture conditions are observed for the cracks of micro and nano sizes.

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References

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