THREE-DIMENSIONAL PROBLEM OF THE CONTACT BY DOUBLY CONNECTED DOMAIN TAKING INTO ACCOUNT ROUGHNESS AND FRICTION

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Summary Expansion of simple fibre potential type integrals distributed on doubly-connected (and simply-connected) domain is received. The method of reduction to the sequence of problems for annular (and circular) contact domains is found out for three-dimensional contact problems with doubly-connected punch under nonlinear laws of friction, roughness of elastic half space.

Introduction

Different statements of contact problems taking into account roughness surfaces, friction and adhesion, for bodies with coating, etc. arose as a result of real-life characteristics consideration. Because of awkward mathematic difficulties in solving of specific real problems, there is a certain "distance" between theory and practical usage of the solutions.

Clear, effective, computational convenient methods are used in [1] for solving circular punch problem problems taking into account roughness surfaces, friction etc. [1,2].

Analogous methods for solving the problem about pressing of doubly-connected punch into elastic half-space under nonlinear friction and roughness laws are developed at present work.

Statement of the Problem

The problem about pressing of close to ring in plan punch into elastic half-space is in consideration. The problem reduced to the solution of the system of equilibrium equations and equation containing integrals with weak singularity.

$$\varphi(p(\rho_0,\theta_0)) + \iint_\Omega \lambda p(\rho,\theta)/r \, d\Omega + \iint_\Omega \cos \widehat{r,x}\, \psi(p(\rho,\theta))/r \, d\Omega = f(\rho_0,\theta_0)\,, \qquad (1)$$

where $\varphi(p(\rho,\theta))$, $\psi(p(\rho,\theta))$, $p(\rho,\theta)$ - functions characterizing deformable properties of roughness, friction law and pressure distribution, correspondingly [1], $r^2 = \rho^2 + \rho_o^2 - 2\rho\rho_o\cos(\theta-\theta_o)$

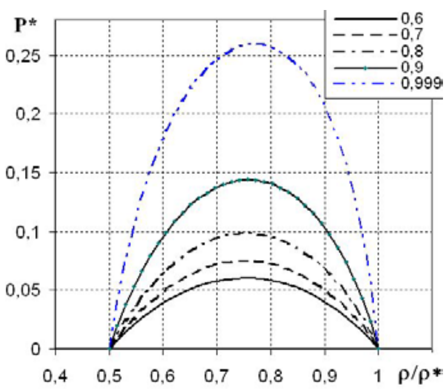

Figure 1

Solution of the Problem for Annular Punch

Equation (1) is transformed to one-dimensional by the proposed in [3] method for the annular punch. Then direct method of successive iterations is used to solve the received equation of second kind. If we receive equation of first kind, regularization with parameter change is used.

Calculation method for the integral and norm of operator is used to specify parameter change interval, conditions of existence and unique solution. With help of potential expansion we assume that analytic in ring function, appropriated to desired pressure function, has singular point in zero relatively analytic extension of regular circular element of the function along any continuous curve through this point. Desired pressure function is presented in view of double series in terms of powers of $\ln\rho$, ρ. Infinite systems of linear algebraic equations are received by substitution of the double series at equation (1). The systems could be solved by method of successive iterations, reduction, exactly, etc.

In case without friction and roughness, among the systems there are homogeneous with triangular matrixes, that have unique trivial zero solution. Only coefficients at plus even and minus odd powers of ρ are nonzero in the double series, they are exactly determined for any index.

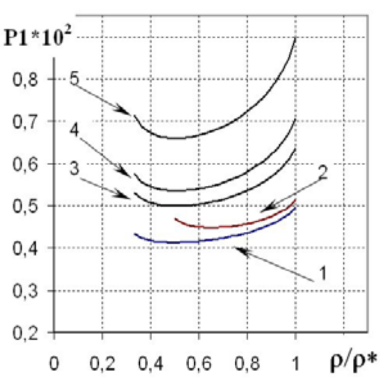

Figure 2

When in (1) $\psi(p(\rho,\theta)) = 0$, $\varphi(p(\rho,\theta)) = B_1 p(\rho,\theta)$ or $B_1 = (1-\alpha)$, fig. 1 shows curves of normal pressure distribution for non-plain parabolic annular punch, values of dimensionless parameters are $\alpha = 0.6, 0.7, 0.8, 0.9, 0.999$, $a/b = 0.5$, $D = 0.5$ - parameter, characterizing geometry of the punch. When α is close to 1 ($B_1 \approx 0$), we receive approximate pressure distribution under smooth punch without roughness of the elastic half-space [3].

We come to nonlinear Hammerstein integral equation in case of nonlinear law of roughness, in particular powers law $\varphi(p(\rho,\theta)) = B_1 p^k(\rho,\theta)$, when $\psi = 0$.

Graphs of contact pressure distribution under annular plain punch are presented on fig. 2 in case of nonlinear power dependence of micro asperity vertical displacement from normal pressure. Curve 1 has next values of parameters: $a/b = 1/3$, $\eta = 0.1$, $B_1 = 0.7$, $k = 0.4$. Next curves differ from curve 1: 2 - $a/b = 1/2$, 3 - $B_1 = 0{,}6$, 4 - $k = 0.45$, 5 - $\eta = 0.14$.

Solution of the Problem for Doubly-Connected Punch

The problem for doubly-connected punch is reduced to the sequence of problems for annular punch [4]. In every approximation, with use of expansion of potential with nonsymmetrical density [5], integral operators are transformed to operators with the same singularity and operators differentiable by Frechet. The problems are reduced to linear in every approximation besides zero approximation in case of nonlinear laws of roughness. Expansion of nonlinear integral operator by parameter is received upon continuous Frechet differentiability of abstract implicit function (Hammerstein operator), when the limits of integration depend from the parameter. Fig. 3 shows the surface of pressure distribution under doubly-connected plain punch close to rectangular ring in plan in case of linear dependence of roughness from pressure, taking into account two approximations, $\alpha = 0.6$, $a/b = 1/2$, $\varepsilon = 0.1406$.

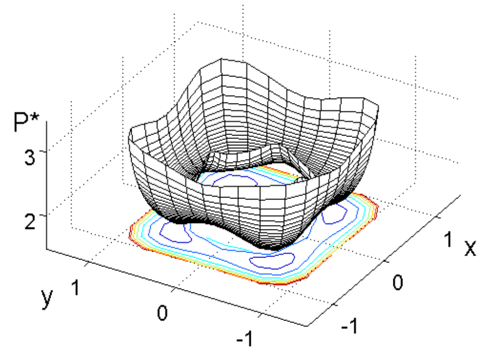

Figure 3

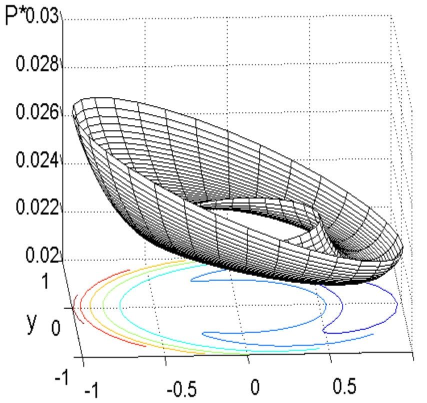

Figure 4

Solution of the Problem Taking into Account Friction

Equation (1) is reduced to one-dimensional with use of expansion of integral with weak singularity proposed in [6,7]. After the transformation of integral operators' kernels to Frechet differentiable, we define regions of parameter variation where the equation has unique solution. The solution could be found out by successive iterations method.

The surfaces of pressure distribution under annular (fig. 4) and circular (fig.5) plain punch are shown in case of nonlinear friction law and without rotation and horizontal diplacements of the punch. Here, condition of neglecting of vertical displacement of micro asperity, resulting from tangential force, takes place. The punch is loaded by vertical and horizontal forces; the last is balanced by friction force. Fig. 6 shows the surface of pressure distribution at the imbalance moment for plain circular punch (linear friction). When α is close to 1 ($B_1 \approx 0$) accounting friction, we receive approximate pressure distribution under the punch without roughness. Here, in case of linear friction the analytical solution is developed, which sensibly coincides with the solution by the method of successive iterations.

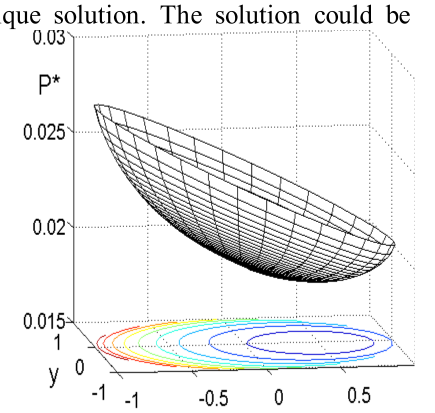

Figure 5

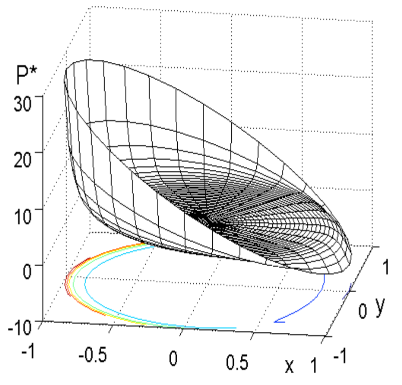

Figure 6

CONCLUSIONS

The method is developed for approximate and exact solutions of integral equations with weak singularity for contact problems with doubly-connected domains taking into account friction and roughness. The method gives possibility to reduce two-dimensional integral equations with weak singularity to one-dimensional. After proposed kernels transformations the reduced integral equations could be solved by approximate computational and analytical methods.

References

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