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WAVE INTERACTION RESONANCES IN INHOMOGENEOUS ELASTIC MATERIALS

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Summary Weakly inhomogeneous nonlinear elastic material is considered. Nonlinear interaction of two counter-propagating longitudinal waves is studied theoretically. An analytical solution is derived. Oscillations on the material boundaries are investigated and wave interaction resonances are clarified. This resonance is sensitive to the properties of the material and to the excitation frequency. This phenomenon is proposed to used for qualitative and quantitative nondestructive characterization of the material.

INTRODUCTION

Wave-wave and wave-material interaction is under investigation due to promising applications in nondestructive testing (NDT) of materials[1, 2]. Utilization of nonlinear effects of wave motion extends the possibilities of NDT [3, 4].
In this paper counter-propagation of two sine waves in weakly inhomogeneous nonlinear elastic material is studied theoretically. An analytical solution is derived for the initial stage of nonlinear wave interaction process. On the basis of this cumbersome solution numerical computations are carried out. Oscillations on the material boundaries are investigated. Wave interaction resonances are clarified. It cleared up that the value of the resonance is sensitive to the properties of the material and to the excitation frequency. Resonance data may be used for qualitative and quantitative nondestructive characterization of inhomogeneous nonlinear elastic material. The last conclusion is supported by the results of numerical experiments.

PROBLEM FORMULATION

A nonlinear elastic material with weak physical inhomogeneity is considered. Deformations of the material are described by the five coefficient nonlinear theory of elasticity [5]. In the one dimensional case the wave process is governed by the equation of motion [6]

$$[\,1 + k_1(X)\,U_{,X}(\,X,t)\,]\;U_{,XX}(\,X,t) + k_2(X)\,U_{,X}(\,X,t) +\; k_3(X)\,[U_{,X}(\,X,t)]^2 - k_4(X)\,U_{,tt}(\,X,t) = 0\,, \qquad (1)$$

where U denotes displacement, X the Lagrangian space coordinate and t the time. The commas and the variables in the indices indicate partial derivations with respect to corresponding variables. The coefficients $k_i(X),\ i = 1\dots4$ are functions of variable properties of the material: density $\rho(X)$, second order elastic coefficients $\lambda(X)$ and $\mu(X)$ and the third order elastic coefficients $\nu_1(X),\ \nu_2(X)$ and $\nu_3(X)$. In the one dimensional case elastic coefficients are grouped. The coefficient $\alpha(X) = \lambda(X) + 2\,\mu(X)$ characterizes linear elastic properties of the material and the coefficient $\beta(X) = 2\,[\nu_1(X) + \nu_2(X) + \nu_3(X)]$ the nonlinear elastic properties. Inhomogeneous material properties ($\gamma = \rho,\ \alpha,\ \beta,\ \varepsilon \ll 1$)

$$\gamma(X) = \gamma^{(1)} + \varepsilon\,\gamma^{(2)}(X),\ \gamma^{(2)}(X) = \gamma_{1\xi}X + \gamma_{2\xi}X^2 + \gamma_{3\xi}X^3,\ \xi = \rho,\ \alpha,\ \beta \qquad (2)$$

have a polynomial functionality.

WAVE INTERACTION

Simultaneous propagation of two waves with arbitrary smooth initial profiles $\varphi(t)$ and $\psi(t)$ are excited on parallel surfaces of the material under initial and boundary conditions $U(X,0) = U_{,t}(X,0) = 0\,,\ U_{,t}(0,t) = \varepsilon\,a_0\,\varphi(t)\,H(t)\,,\ U_{,t}(L,t) = \varepsilon\,a_L\,\psi(t)\,H(t)$, where $H(t)$ is a Heaviside function and constants εa_0 and εa_L depict the initial wave amplitudes.
Eq. (1) with Eq. (2) is solved following the perturbation procedure. The solution is sought in the form of series

$$U(X,t) = \sum_{n=1}^{\infty} \varepsilon^n\,U^{(n)}(X,t)\,, \quad 0 < \varepsilon \ll 1. \qquad (3)$$

The first three terms in solution (3) are derived making use of the software package $Maple$. The first term is the solution to the linear wave equation and it consists of four constituents

$$\begin{aligned} &U^{(1)}_{,t}(X,t) = a_0 H(\chi)\varphi(\chi) + a_L H(\eta)\psi(\eta) - a_0 H(\theta)\varphi(\theta) - a_L H(\zeta)\psi(\zeta)\,,\\ &\chi = t - X/c, \quad \eta = t - (L - X)/c, \quad \theta = t - (2L - X)/c, \quad \zeta = t - (X + L)/c\,. \end{aligned} \qquad (4)$$

Second and subsequent terms are solutions to the PDEs with known r.h.s. Expressions for these terms are too cumbersome to be presented here. They correct the linear solution and take the nonlinear effects of wave motion and material inhomogeneity into account.
The derived solution is valid in the interval $0 \leq t\,c/L < 2$, where c is the phase velocity.

WAVE INTERACTION RESONANCES FOR MATERIAL CHARACTERIZATION

For applications, the excited wave profiles are specified by sine functions $\varphi(t) = \psi(t) = \sin(\omega\, t)$, where ω is the radial frequency, i.e., simultaneous propagation, reflection and interaction of harmonic waves is considered. The corresponding cumbersome solution is derived and analyzed numerically. The material is assumed to be duralumin with constant part of the density $\rho^{(1)} = 3000$ kg/m^3, the elastic coefficients $\alpha^{(1)} = 100$ GPa and $\beta^{(1)} = -750$ GPa. Material has two parallel boundaries and the thickness $L = 0.1$ m. The following notation for the parameters of inhomogeneity is used:

$$\gamma(X) = \gamma^{(1)}\,(1+\delta_{i\xi}(X)), \quad \delta_{i\xi}(X) = \varepsilon\gamma_{i\xi}X^i/\gamma^{(1)}, \quad \delta_{i\xi}(L) \equiv \delta_{i\xi}, \quad i = 1,2,3, \quad \gamma,\xi = \rho,\alpha,\beta. \tag{5}$$

Wave processes is excited in the range of the values of excitation amplitudes $-1.30\,c \le a_0 \le -0.35\,c$, $a_L = -a_0$ m/s and $\varepsilon = 10^{-4}$. Some of the computed twenty values of the boundary oscillation amplitudes $A_{li},\ i = 1\ldots 20$ are plotted in Fig 1 for the linear case. The maximum (resonant) amplitude in the interaction interval $1 < t\,c/L < 2$ occurs always at the same frequency $\omega_l = 2\,\pi\,c\,n/L$ where n is an integer.

Results of the numerical simulation of the resonance in the interaction interval on the basis of the whole nonlinear solution are presented in Fig.2. Amplitudes A and frequencies ω are computed in Fig. 2 for the first oscillation peak by $n = 4$. It becomes clear that the wave interaction resonance is essentially sensitive to the values of material properties and to the inhomogeneity in density and linear elastic properties. The weak inhomogeneity in nonlinear elastic coefficient $\beta(X)$ practically does not affect on the value of the resonance. Results of the numerical simulation compose cascades in different regions in Fig. 2. These cascades form different *corridors* for linear, quadratic and cubic inhomogeneities in material properties. This is illustrated for $\alpha(X)$ in Fig. 2. Important is that inhomogeneity in density has essentially different effect on wave interaction resonance than other material properties (see Fig. 2). This makes it possible to use the obtained results in ultrasonic nondestructive evaluation of the properties of inhomogeneous materials.

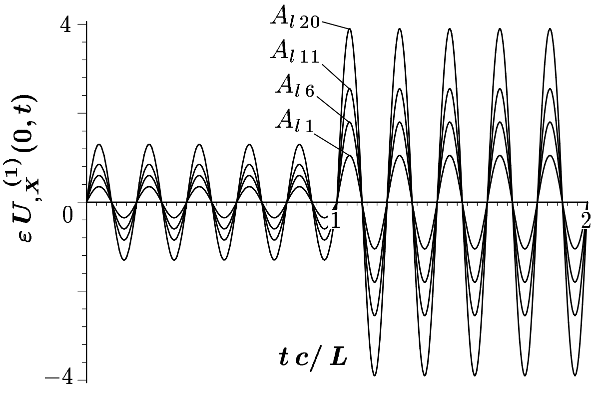

Figure 1. Variation of the boundary oscillation amplitude for the linear case $(n = 5)$.

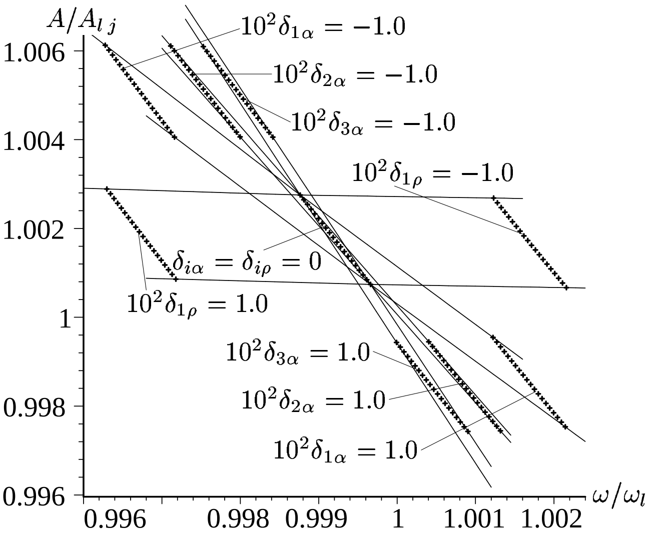

Figure 2. Nonlinear resonance as a function of the frequency and material properties $(j = 1\ldots 20,\ i = 1,2,3)$.

CONCLUSIONS

Two wave interaction resonance in the inhomogeneous elastic material is studied theoretically. This resonance is sensitive to the inhomogeneous material properties and it may be used in NDT. The work is supported by ESF through NATEMIS program and by Estonian SF through grant no. 4706.

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