XXI ICTAM, 15-21 August 2004, Warsaw, Poland

NEW CLASSES OF ANALYTICALLY DERIVED OPTIMAL TOPOLOGIES AND THEIR NUMERICAL CONFIRMATION

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Summary New classes of exact, analytically derived structural topologies are outlined, their underlying theory summarised, illustrative examples given and their numerical, FE-based confirmation by the DCOC/SIMP method presented..

INTRODUCTION

Topology optimization in real world problems requires highly efficient, numerical (FE-based) methods, which can easily lead to grossly erroneous solutions due to discretization errors (e.g. checkerboard patterns, diagonal element chains, and hinges in plates), singularity of the topology, local minima and other factors. Closed form, exact analytical representation of the optimal topology is therefore very useful in checking on the validity, accuracy and convergence of numerical methods. Analytical solutions are also essential in understanding the true nature and fundamental properties of optimal topologies. It is rather unfortunate that in our time very few researchers carry out research into the theory of exact optimal topologies.

In this paper, the theory of several new classes of optimal topologies is briefly outlined, their application to particular problems illustrated with examples and the analytical solutions confirmed by numerical, FE-based computations.

OVERVIEW OF NEW CLASSES OF ANALYTICALLY DERIVED TOPOLOGIES

It is well known that optimization of perforated plates in in-plane stress leads to the same topology as that of plane trusses, if the volume fraction of the former tends to zero. Exact analytical solutions for optimal truss topologies are therefore used regularly for verifying numerical solutions for perforated plates in plane stress.

(a) Exact optimal topology of new classes of trusses with a stress constraint

In this section, the classical Michell theory [1] is applied to new classes of boundary conditions.

It was already mentioned at ICTAM 2000 by the first author that the exact Michell solution is not yet known for even quite simple boundary and support conditions, particularly for point supports. Michell's *optimality conditions* require that the „adjoint" strain has a constant (say k) absolute value in all truss members and it does not exceed this value in any direction at any point of the design domain for the truss. For zero reaction cost, the adjoint displacements at supports are zero. The Michell problem is convex and even selfadjoint (the real and adjoint strains are proportional). Exact optimal topologies for point-supported cantilevers (including a half „MBB-beam") were given earlier [2] by the first author's research group. New solutions are presented in this paper.

(b) Different permissible stresses in tension and compression

For these problems the adjoint strains are equal to the reciprocal values of the permissible stresses along members (compression is negative) and can take on any value in between the above limits anywhere else within the design domain. This problem is convex but nonselfadjoint. It was explained earlier by the first author [3], that Michell [1] made certain mistakes in deriving his (incorrect) optimality conditions for unequal permissible stresses in tension and compression. The consequences of this error can be quite severe in terms of truss weight [3]. Solutions for this class of problems consist of orthogonal Hencky nets, but the optimal member directions are in general not at 45 degrees to the supporting lines.

(c) Variable external forces (including reactions) of non-zero cost

This problem class has application in support and ballast optimization and in passive control. For these problems the above optimality conditions apply, but at supports (or variable external forces) the adjoint displacement is the subgradient of the support cost function with respect to the reaction force (variable force). If the support cost is r-times the absolute value of the support force (uplift is negative), then the adjoint displacement is r for positive and –r for negative reaction. For zero reaction (variable force) the adjoint displacement may take on any value in between –r and r. This problem class can be made convex and selfadjoint.

(d) Optimization for one or several active displacement constraints

In this case the adjoint strains are proportional to the sum of the products of Lagrange multipliers and virtual strains and the optimal cross sectional areas are proportional to the square root of the sum of the products of the real and adjoint strains. The problem is in general nonconvex and nonselfadjoint. The topology for this class of problems consists of „generalized" Hencky nets, in which the lines representing optimal member centre lines are in general non-orthogonal.

(e) Optimal topologies for external load plus self weight

It was shown by the first author ([7], see also [8], pp. 72-73 and 206-218) that the adjoint strain for the above structures is the usual adjoint strain multiplied by (1+u), where u is the adjoint displacement in the direction of the gravitational forces (in usual terrestrial structures the vertical direction). This problem class is nonconvex and nonselfadjoint.

Problem class (a) as a special case of problem classes (b), (c), d) and (e).

If under (b) the two permissible stresses are equal, under (c) the reaction costs are zero, under (d) the displacement constraint is a compliance constraint, and under (e) the specific truss weight tends to zero, then the latter four reduce to problem class (a). This can be used for checking on the validity of more advanced solutions.

EXAMPLES OF EXACT SOLUTIONS

At least two examples for non-elementary boundary and support conditions will be derived analytically for each of the above classes of problems, giving the full details of the derivation. The examples will include line and point supports.

NUMERICAL CONFIRMATION OF THE ANALYTICAL RESULTS

The analytical solutions will be confirmed by FE-based numerical computations using the now universally employed SIMP algorithm [4, see also derivation in 5], and its extensions using DCOC [6]. It is to be mentioned that SIMP was first implemented by the first author's research group in the late eighties, when most others were strongly debating the validity and usefulness of this method

Acknowledgement

The present study was conducted in the research group of Computational Structural Mechanics of the Hungarian Acad. Sci. and was supported by The Hungarian National Scientific and Research Foundation (OTKA) (grant T 037922).

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