OPTIMAL STRUCTURES FOR BUCKLING FORCES AND BUCKLING DISPLACEMENTS

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Summary In the present paper the problems of optimal design of elastic columns and annular plates, against buckling are investigated. The optimal solutions for loadings controlled by forces and by displacements are presented and compared in the paper. The unimodal and bimodal problems are considered. The results are obtained using the numerical optimization methods, namely the Method of Moving Asymptotes and the Simulated Annealing Method

INTRODUCTION

The optimal design of structures under stability constraints concerns mainly loadings controlled by a system of forces. However, in some practical engineering applications, the loadings, which are controlled by displacements, can also occur. This type of problems is, for example, connected with structures under thermal loadings in the case of immovable supports or in the case of assembly loadings, which are due to initial imperfections of overall dimensions of a structure. Then, the compressive forces, which occur due to assembly displacements or due to an elevated temperature, depend on geometry of the structure, whereas in the classical optimization problem the forces are independent of the structure. Hence, the results of shape optimization can be qualitatively and quantitatively different for both types of loading control.

Relatively many papers deal with optimal design of structures under stability constraints, taking into account loadings controlled by forces. On the other hand, the problem of optimization of structures against buckling under loadings controlled by displacements was investigated, for example, in the papers by Albul, Banichuk, Barsuk [1], Krużelecki, Smaś [2, 3].

The present paper shows a comparison of the optimal shapes of columns and annular plates obtained for loadings controlled by displacements and by forces. The unimodal and multiodal formulations of the optimization problem are applied.

FORMULATION OF THE OPTIMIZATION PROBLEM

We consider an elastic structure on an elastic foundation of the Winkler type with a modulus β. A compressive force or a displacement of the edge loads the structure. It is assumed that both, the force resulting from the displacement of the edge and the compressive force are conservative ones. We look for such a distribution of the material of a structure, which leads to the maximal critical loading (force or displacement), before the structure buckles.

The optimization problem is stated under equality and inequality constraints. It is assumed that the optimal structure has the same volume of material V as a reference structure volume V_0 with a uniform distribution of material. Additionally, the minimal and maximal values of the design variables are constrained. In a case of a multimodal optimization problem, the condition of equality of the lowest (two or more) critical forces is also imposed.

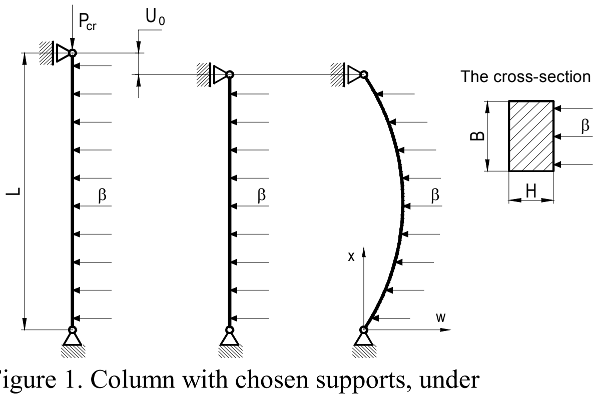

Figure 1. Column with chosen supports, under loadings controlled by displacement U_0 or force P_{cr}

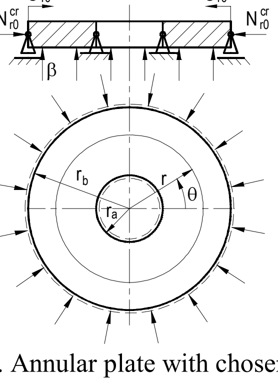

Figure 2. Annular plate with chosen supports, under loadings controlled by displacement U_{r0} or force N_{r0}^{cr}

Optimization of a column of length L with a rectangular cross section of width B and depth H, on the Winkler foundation, supported by a general system of supports and loaded by a compressive force P_{cr} or axial displacement U_0 is considered as one-dimensional problem. A chosen case of supports is shown in Fig.1. The cross-sectional area $A(x)$,

varying along the axis of the column is a design variable. Furthermore, the relation between the cross-sectional area *A* and moment of inertia *I* is taken in the following form: $I = cA^n$, where *c* is a certain constant and *n=1* is for the cross-sections of constant depth and variable width, *n=2* for geometrically similar cross-sections or *n=3* for cross-sections of constant width and variable depth.
However, two-dimensional problem is represented by the optimization of annular plate. A thickness *H(r)* of the plate that is design variable, is assumed as circularly symmetric and it can vary only in radial direction. The outer or inner edge of the plate can be loaded by radial force N_{r0}^{cr} or radial displacement U_{r0}. The annular plate loaded at the outer edge with a chosen support is shown in Fig.2.

NUMERICAL RESULTS

For the numerical formulations of the optimzation problems stated above two different optimization methods, namely the Method of Moving Asymptotes (MMA) and the Simulated Annealing Method (SAM) were applied. These optimization methods were used for one- and two-dimensional problems. Both methods, allowing for the equality and inequality constraints, were used either separately or alternately, depending on the investigated problem.
The optimal distributions of the cross-sectional area for simply supported column of constant depth (n=1) on an elastic foundation with dimensionless modulus $\overline{\beta} = 100$ are shown in Fig.3. The results are obtained for different types of loading, namely loading controlled by displacement (solid line) and by force (dashed line). The profit measured by increasing of critical loadings, related to prismatic column equals about 78 per cent for loading controlled by displacement and 20 per cent for loading controlled by force.
The optimal shapes of annular plate for both case of loading are presented in Fig.4. The shape plotted by thick line corresponds to loading controlled by displacement whereas the thin line relates to loading controlled by force. The elastic foundation is not active. The optimization was carried out using SAM method, 48 design variables were applied. The obtained results (Fig.4) are unimodal for both types of loading. The increase of critical loadings, in relation to plate of constant thickness, amount about 67 per cent for critical displacement of the edge and 47 per cent for critical force.

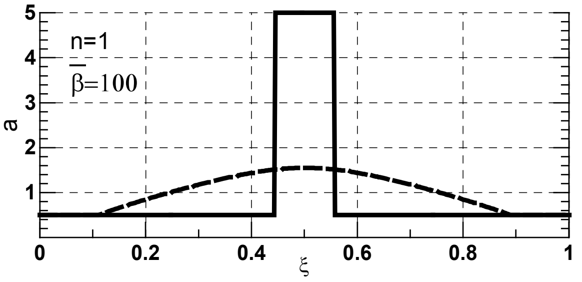

Figure 3. Distribution of dimensionless cross-sectional area *a(ξ)* of simply supported column on an elastic foundation

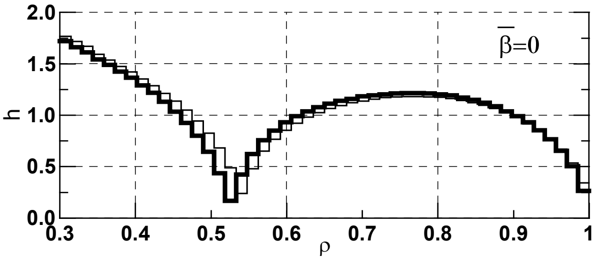

Figure 4. Distribution of dimensionless thickness *h(ρ)* of annular plate with immovably clumped inner edge, movably simply supported and loaded outer edge.

CONCLUSIONS

The results presented in the Fig.3 show that shapes of columns for both types of ladings are completely different. The differences depend on values of geometrical constraints and value of coefficient $\overline{\beta}$ of an elastic foundation. The lower bound imposed on cross-sectional area of the column has to be necessary in a case of loading controlled by displacements whereas the upper bound constraint seems to be necessary only for a certain group of optimization problems.
The results obtained for annular plates (Fig.4) show very small differences in the optimal shapes and critical loadings for both types of loadings.

References

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