XXI ICTAM, 15-21 August 2004, Warsaw, Poland

DISSIPATION FEATURES AT NONLINEAR PULSATIONS OF BUBBLES IN VISCOELASTIC FLUIDS

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Summary Nonlinear interaction of heat-conducting gas bubble with viscoelastic liquid in a sound field of small but final amplitude is described. The liquid phase is treated as non-Newtonian fluid following Oldroyd type rheological equation. Solution of the problem is received within the volume approach in quadratic approximation with respect to the incident wave amplitude. Resulting relation for the scattered wave intensity is studied numerically with the emphasis on the dissipation features. The study is motivated by the problem of acoustic control of microbubbles trapping in flows of smart fluids with memory.

INTRODUCTION

Microbubbles can change drastically dynamic properties of the liquid and therefore their appearance in smart fluids flows must be controlled carefully. One of perspective tools for microbubbles diagnostics is the acoustic method based on high scattering ability of small inclusions of free gas in a liquid. Among different modifications of acoustic method, the nonlinear ones attract special attention because allow to detect bubbles already at low bubble concentrations and sufficiently small amplitudes of the fundamental wave [1]. It is explained by the fact that nonlinearity of gas within compressible bubble is responsible for the major input in nonlinear properties of the gas-liquid mixture [2]. Small gas bubble in a sound field of final amplitude can be treated as nonlinear oscillator generating the scattered signal at multiple frequencies along with the basic one. The theory of second harmonics generation by small bubbles has been developed in [2] and it was shown that the amplitude of this harmonic is sufficiently large to be registered [3]. The goal of the present study is to generalize this theory for the case of viscoelastic liquid. The volume approach [2] is used here, but, as distinct to the previous studies [2-4], the liquid nonlinearity is taken into account, the losses are not supposed to be additive and are calculated consecutively within the general perturbation scheme.

FORMULATION OF THE MODEL

Hydrodynamic problem

It is supposed that rheological properties of the liquid can be described by Oldroyd type rheological equation:

$$\tau = \tau^{(1)} + \tau^{(2)}, \qquad \tau^{(1)} + \lambda[D\tau^{(1)}/Dt - \alpha(\tau^{(1)}\cdot e + e\cdot\tau^{(1)})] = 2\eta\beta e, \qquad \tau^{(2)} = 2\eta(1-\beta)e \tag{1}$$

Here τ, ω and e are deviator of the stress tensor, vorticity and rate deformation tensors, respectively; λ - relaxation time; η - Newtonian viscosity of the liquid; β - parameter, characterizing input of the Maxwell element in the effective viscosity of the liquid ($0 \le \beta \le 1$); α - parameter of the liquid nonlinearity; D/Dt - Jaumann derivative. The generalized Raleigh equation for small spherical bubble, oscillating in a sound field, is formulated in suggestions that the bubble can be considered as a monopole scatterer (the sound wavelength is much larger than the equilibrium bubble radius R_0), the liquid is incompressible and its rheology, the same as in the case of a purely viscous liquid, is manifested only in the close vicinity of the interface. The resulting integro-differential equation for the bubble radius $R = R(t)$ can be reduced to two coupled differential equations:

$$\rho(R\ddot{R} + \tfrac{3}{2}\dot{R}^2) = \Delta p_g - p_a + \frac{2\sigma}{R_0}(1-\frac{R_0}{R}) - 4\eta(1-\beta)\frac{\dot{R}}{R} + \tau_R, \qquad \tau_R(1+4\alpha\lambda\frac{\dot{R}}{R}) + \lambda\frac{\partial\tau_R}{\partial t} = -4\eta\beta\frac{\dot{R}}{R} \tag{2}$$

Here ρ, σ are density and surface tension coefficient of the liquid; $\Delta p_g = p_g - p_{g0}$, $p_{g0} = p_0 + 2\sigma R_0^{-1}$, where p_0, p_g are equilibrium pressure in the liquid and current pressure in the gas phase; p_a - pressure disturbance in the incident wave with frequency ω. The far-field component of the pressure in the scattered wave has the form (rheology of the liquid influences the scattered signal through the bubble dynamics equation only):

$$p_S(r,t) = \rho\left(R^2(t_r)\ddot{R}(t_r) + 2R(t_r)\dot{R}^2(t_r)\right)r^{-1}, \qquad t_r = t - r/c_0, \tag{3}$$

where c_0 is the sound speed in the liquid, r is the radial coordinate and t_r - the retarded time [3].

Gas dynamic problem and boundary conditions

The pressure within pulsating gas bubble in a wide range of conditions doesn't depend from the radial coordinate and therefore the internal problem can be written in the following form:

$$\rho_{g0}c_{pg}\left(\frac{\partial T_g}{\partial t}+v_g\frac{\partial T_g}{\partial r}\right)=k_g\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_g}{\partial r}\right)+\frac{dp_g}{dt},\qquad \frac{dp_g}{dt}=-\frac{3(\gamma_g-1)}{R}q_R-\frac{3\gamma_g}{R}p_g\frac{dR}{dt} \tag{4}$$

$$q_R=-k_g\left(\frac{\partial T_g}{\partial r}\right)_{r=R(t)},\qquad \frac{\partial\rho_g}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho_g r^2 v_g)=0,\qquad p_g=\rho_g T_g c_{vg}(\gamma_g-1),\quad \gamma_g=c_{pg}/c_{vg}$$

Here T_g, ρ_g, v_g are gas temperature, density and the radial velocity in gas; k_g, c_{pg}, c_{vg} - heat conductivity and specific heat capacities of gas. The boundary conditions at $r=R(t)$ mean: $T_g=T_0$, $v=\dot{R}(t)$. At the bubble centre $\partial T_g/\partial r=0,\ v_g=0$.

NONLINEAR SCATTERING OF THE INCIDENT WAVE

Following the volume displacement approach [2, 3], the disturbance of the bubble volume ΔV is introduced in (2)-(4) instead of the radius R ($\Delta V=(4\pi/3)(R^3-R_0^3)$). The result is written in dimensionless form, keeping all the terms up to ΔV^2. Solution of the problem is searched in the form:

$$\{\delta\tilde{V},\ \delta\tilde{p}_s,\ \delta\tilde{u},\ \delta\tilde{p}_g,\ \delta\tilde{\rho}_g,\ \delta\tilde{v}_g,\ \delta\tilde{T}_g\}=\{\delta\tilde{V}_1,\ \delta\tilde{p}_{s1},\ \delta\tilde{u}_1,\ \delta\tilde{p}_{g1},\ \delta\tilde{\rho}_{g1},\ \delta\tilde{v}_{g1},\ \delta\tilde{T}_{g1}\}+ \tag{5}$$

$$+\{\delta\tilde{V}_2,\ \delta\tilde{p}_{s2},\ \delta\tilde{u}_2,\ \delta\tilde{p}_{g2},\ \delta\tilde{\rho}_{g2},\ \delta\tilde{v}_{g2},\ \delta\tilde{T}_{g2}\},\qquad \tilde{p}_a=\frac{1}{2}\tilde{P}_a e^{i\tilde{\omega}\tilde{t}}+c.c.,$$

where $\delta\tilde{V}$, $\delta\tilde{p}_s$, $\delta\tilde{u}$, $\delta\tilde{p}_g$, $\delta\tilde{\rho}_g$, $\delta\tilde{v}_g$, $\delta\tilde{T}_g$ are dimensionless disturbances of the bubble volume, pressure in the scattered wave, radial component of the stress tensor deviator in liquid at the interface, gas pressure and density, velocity and temperature in the gas phase, respectively. Vector components with the subscript 1 have the order of $\tilde{P}_a$ and are proportional to $e^{i\tilde{\omega}\tilde{t}}$ while that with the subscript 2 have the order of $\tilde{P}_a^2$ and are proportional to $e^{2i\tilde{\omega}\tilde{t}}$. The scaling is done with the use of equilibrium parameters R_0, p_0, ρ, T_0. A tilde is used to distinguish dimensional variables from their nondimensionalized equivalents. The equations governing the pressure in the linear scattered wave and the amplitude of the second harmonics are obtained and solved consecutively. The resulting relations are analyzed and the basic features of nonlinear wave scattering by small gas bubble in viscoelastic liquid are established.

CONCLUSIONS

The thermal and rheological losses in viscoelastic liquid at linear wave scattering can be treated as additive factors the same as in a pure viscous liquid [5]. This result isn't hold for the signal scattered at the second harmonics frequency, that is explained by the losses coupling. The influence of rheological nonlinearity on the amplitude of the second harmonics grows with the amplitude of the incident wave. Nevertheless, in a wide frequency region the gas nonlinearity is the dominant factor; the features of liquid nonlinearity are manifested mainly in the vicinity of the linear resonance ($\omega\sim\omega_0$). Scattering cross section of the bubble lowers with growth of the liquid viscosity η both for basic and second harmonics as a result of increase in rheological losses. Relative amplitude of the second harmonics (with respect to the basic one) for viscoelastic liquid is always greater than for similar pure viscous liquid because of frequency-dependent behavior of dynamic viscosity. The results of simulations indicate that even for sufficiently viscous smart viscoelastic liquids the amplitude of nonlinear scattered wave at the resonance is large enough to be detected by the same acoustic equipment that is used for bubbles diagnostics in water. It opens possibilities for acoustic control of bubbles trapping in flows of viscoelastic smart liquids with high viscosity in the same manner as for low viscous Newtonian fluids.

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