BIFURCATION BUCKLING OF SANDWICH PLATES AND SHELLS IN PLASTIC RANGE

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Summary Bifurcation buckling of complete circular sandwich plates and complete spherical sandwich shells subjected to uniform equi-biaxial compression is investigated when faces undergo plastic straining and core remains elastic. The presented theory accounts for transverse shear strains. Both J_2 incremental and J_2 deformation theories of plasticity are used. The results generalize the classical formulas for elastic and plastic buckling by adding shear correction terms.

INTRODUCTION

Plastic buckling has a long history of paradoxes starting with the well-known column paradox [1]. It was resolved by Shanley by showing that bifurcation in plastic range occurs with increasing load. A new paradox arose in connection with the plastic buckling of axially compressed rectangular plates. The Mises or J_2 incremental theory of plasticity, accepted as a correct phenomenological theory, predicts bifurcation loads for such plates much higher than experiments. Paradoxically, the incorrect, J_2 deformation theory of plasticity yields good results. This paradox was explained by Onat and Drucker by demonstrating the extreme sensitivity of the incremental theory to geometric imperfections. For axially compressed cylindrical shells, the paradox was found by Batterman to be not as dramatic.

Bijlaard [2] was among the first investigators to conduct rational analyses of plastic buckling of plates and shells, including sandwich plates. However, he argued that the deformation theory was the correct theory. The author has tackled plastic bifurcation problems [3,4,5] by employing both plasticity theories and by including transverse shear strains. The contention is that the two theories need not give widely differing bifurcation loads for all cases.

As interesting examples, the paper presents general non-axisymmetric buckling of complete circular plates and complete spherical shells of sandwich type with equal face thicknesses, subjected to uniform equi-biaxial compression. The loading consists of peripheral radial pressure for plates, and external pressure for shells. Transverse shear strains are included by introducing rotations of normals as additional kinematic variables. Only faces are allowed to undergo plastic straining; core remains elastic. Results generalize previous formulas [2,6,7] by supplying shear correction terms.

OUTLINE OF THE ANALYSIS AND SOME RESULTS

General constitutive relations

Thin shell assumption is used. Pre-buckling strains are considered constant, though stresses are different in core and faces due to differences in moduli. The constitutive relations below are in a form applicable to elastic (core) or plastic (faces) behaviour. Let $\overset{\circ}{\sigma}$ be the pre-buckling equi-biaxial compressive stresses in faces due to the applied pressure. The Mises yield condition for such a stress-state is $\sigma_{\text{eff}} = \sqrt{3J_2} = \overset{\circ}{\sigma} = \sigma_Y$ where σ_Y is in strain-hardening range of a uniaxial $\sigma - \epsilon$ curve of the face material. Bifurcation induces stress and strain increments, σ_{ij} and ϵ_{ij}, related by the applicable constitutive relations. For the usual small strain plasticity, these relations for J_2 deformation theory [2-5] are

$$\sigma_{11} = B'\epsilon_{11} + C'\epsilon_{22},\ \sigma_{22} = C'\epsilon_{11} + B'\epsilon_{22}, \sigma_{33} = 0,\ \sigma_{ij} = 2F'\epsilon_{ij}\,(i \neq j),\ \text{where}$$

$$B' = A'(3E/E_s + E/E_t), C' = A'(3E/E_s - E/E_t - 2 + 4\nu), F' = \frac{E}{(3E/E_s - 1 + 2\nu)}$$

$$A' = \frac{E}{(E/E_t + 1 - 2\nu)(3E/E_s - 1 + 2\nu)}.$$

Note that $C' = B' - 2F'$. Subscripts 1, 2, 3 refer to radial (or meridional), circumferential, and thickness directions. E and ν are elastic moduli. $E_t = d\sigma/d\epsilon$ and $E_s = \sigma/\epsilon$ are the tangent and secant moduli at stress level $\sigma = \overset{\circ}{\sigma}$ on the $\sigma - \epsilon$ curve. For incremental theory $E_s = E$. For elastic cases $E_s = E_t = E$. Unloading is precluded at plastic bifurcation.

Kinematic assumptions and governing equations

The radial (or meridional), circumferential, and normal displacements arising due to buckling are taken respectively as $u - z\alpha,\ v - z\beta,\ w,$ where z is the thickness coordinate. u and v are the 'inplane' middle surface displacements, w is the 'out-of-plane' displacement, and $\alpha,\ \beta$ are the rotation components of the middle surface normals. Thus, there are five kinematic variables which are functions of the two surface coordinates. For plates, $u = v = 0$, only three variables are present. Buckling strains ϵ_{ij} in polar or spherical coordinate system follow from these displacements. Principle of virtual work for initially stressed bodies yields equations of equilibrium and boundary conditions in terms of stress resultants consistent with the kinematic assumptions. The stress resultants are expressed in terms of the kinematic variables by using the constitutive relations and integrating through core and face thicknesses. Substitution of the stress resultants thus expressed yields the governing equations and boundary conditions in terms of the kinematic variables.

Some results

Solutions are obtained in Bessel functions for the plate buckling, and in associated Legendre functions for the shell buckling. Space limitation prevents giving details, except some for spherical shell. The minimum buckling pressure is

$$q_{cr}(\text{min}) = \frac{4hB_2}{R^2}\sqrt{\frac{4F_1(B_1-F_1)}{B_1B_2}}\{1-(\frac{h}{R})\sqrt{\frac{4F_1(B_1-F_1)}{B_1B_2}}\times\frac{B_2}{2G}\}$$

where $R=$ middle surface radius, $h=$ core thickness, $t=$ face thickness, $f(t)=6(t/h)+12(t/h)^2+8(t/h)^3$,

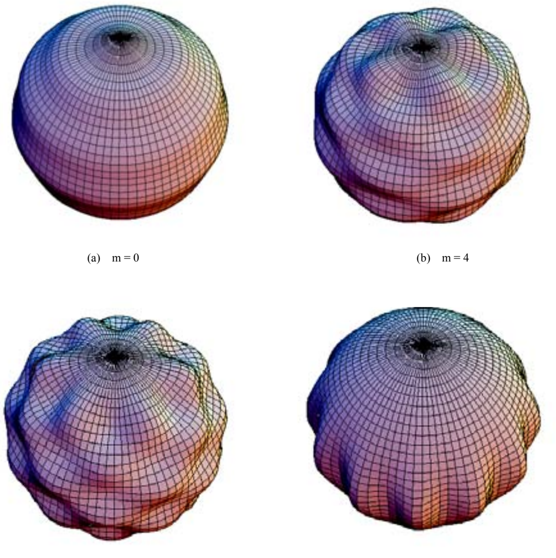
$$B_1 = B'_c h\,\{1+\frac{tB'_f}{hB'_c}\},\ F_1 = F'_c h\,\{1+\frac{tB'_f}{hB'_c}\}, B_2 = \frac{B'_c h}{12}\{1+\frac{B'_f}{B'_c}\,f(t)\}, F_2 = \frac{F'_c h}{12}\{1+\frac{F'_f}{F'_c}\,f(t)\}, G = kF'_c h.$$

Subscripts f and c refer to face and core moduli. k is shear correction factor. The integer n corresponding to q_{cr}(min) is

$$n(n+1)\approx\frac{2F_1}{B_1}+(R/h)\sqrt{\frac{4F_1(B_1-F_1)}{B_1B_2}}\{1+(\frac{h}{R})\sqrt{\frac{4F_1(B_1-F_1)}{B_1B_2}}\times\frac{B_2}{G}\}.$$

n is related to meridional buckles as explained below. Terms containing G in expressions for q_{cr}(min) and n are shear correction terms. Known formulas for homogeneous shell buckling [2,6,7] are obtained by appropriate specialization and $G\to\infty$. Numerical results show that, as usual, the buckling pressure given by the deformation theory is lower than that from the incremental theory. However, the differences remain within 10% for realistic material properties and geometry. Surprisingly, for simply supported circular sandwich plates, such differences are found almost negligible.

A mode shape of the radial displacements is $w=\cos(m\theta)\,P_n^m(\cos\phi)$ where θ and ϕ are circumferntial and meridional coordinates, $P_n^m(\cos\phi)$ are associated Legendre functions, $2m=$ circumferential buckles, $n-m+1=$ meridional buckles. Although n is uniquely determined from the above formula, m is allowed to be any integer from 0 to n, where $m=0$ signifies an axisymmetric mode. Thus, there is a multiplicity of eigenmodes for the same eigenvalue, a familiar phenomenon in shell buckling. Here, the multiplicity has been determined unambiguously by the analysis. The figure below shows four $(m=0,\ 4,\ 6,\ 12)$ of the thirteen modes for a buckling pressure for which $n=12$.



(a) m = 0 (b) m = 4

(c) m = 6 (d) m = 12

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