A STATISTICAL MECHANICS THEORY OF RANDOM HONEYCOMB AND OPEN-CELL FOAM STRUCTURES

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Either by design or otherwise, many engineering materials are randomly structured. Examples include atomistically disordered or partially disordered materials such as bulk metallic glasses or polymers, and macroscopically disordered materials such as foam materials or random grain piles. Because of structural randomness, the internal force distribution in these materials due to external loadings would not be uniform, yet a thorough understanding of the force distribution is of paramount importance in the development of, for example, yield criteria for these materials. In this work, finite element simulations are used to investigate the force distribution in stressed random honeycomb and open-cell foam structures. A statistical-mechanics-based theory is also presented to describe the force distribution.

FINITE ELEMENT SIMULATIONS

2-D honeycombs and 3-D open-cell foam structures were simulated in this work using the FEAP finite element package developed by R.L. Taylor. Fig. 1(a,b) shows a simulated perturbed square grid (2-D) and a cubic grid (3-D). A series of increasingly irregular structures were built from a regular square or cubic grid by randomly displacing each node in the grid within a range which is a certain fraction r_d (< 0.5) of the mean grid spacing. The fraction r_d then becomes a parameter characterising the randomness of the structure. The linear elastic behaviour of the structures under different load states was calculated using the Euler-Beam Element in the FEAP package.

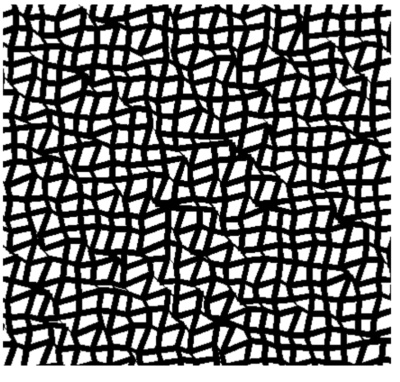

Fig. 1(a) 2-D perturbed square grid (r_d = 0.4).

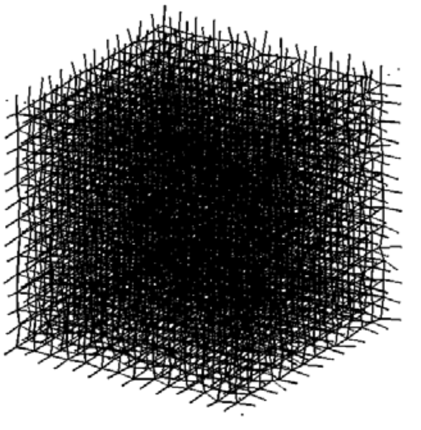

Fig. 1(b) 3-D perturbed cubic grid (r_d = 0.4).

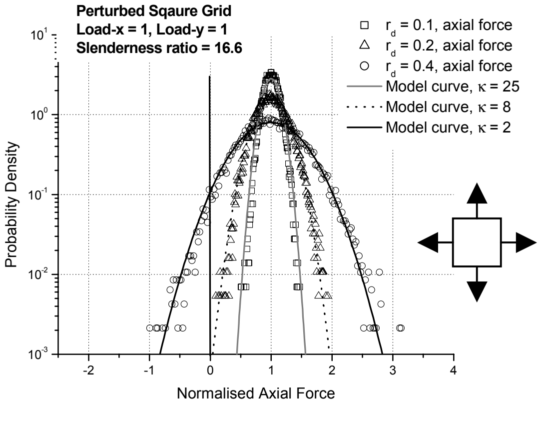

Fig. 2(a) Axial force distributions of 2-D perturbed square grid under hydrostatic loading.

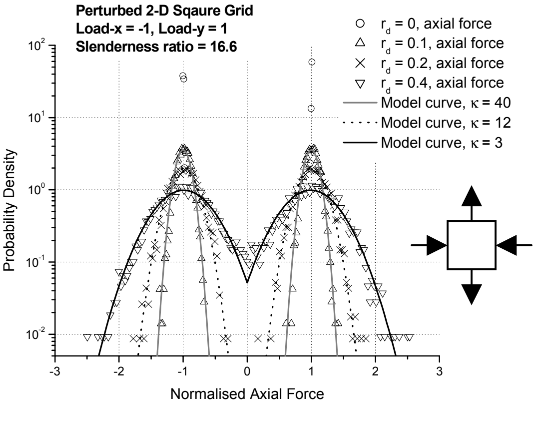

Fig. 2(b) Axial force distributions of 2-D perturbed square grid under biaxial pure shear loading.

Fig. 2(a) shows the simulated axial force distribution in 2-D grids with different values of r_d under hydrostatic loading. It can be seen that the sharpness of the axial force distribution decreases as r_d, or the structural randomness, increases. Fig. 2(b) shows the axial force distributions under biaxial pure shear. At $r_d = 0$, the distribution comprises two delta functions positioned at two (bifurcated) values of the mean force. This is to be expected because in the regular square grid configuration ($r_d = 0$), all struts which are aligned parallel to the tensile axis will be subject to the same tensile force, and all those parallel to the compression axis will be subject to the same compressive force. As r_d increases from zero, the structure becomes increasingly perturbed and the force distribution becomes more spread as shown in fig. 2(b). Results in 3-D show similar features.

THEORY

A theory based on statistical mechanics concepts will be presented to predict the general form of the force distributions shown typically in fig. 2. The key concept in this theory is the introduction of an entropy constraint in the search for the ground state configuration of the structure. Thus, instead of minimising the strain energy U alone as the condition for static equilibrium, we minimise a free energy functional defined as

$$F = U - \theta S \tag{1}$$

where S is an entropy functional and θ is an analog of temperature, but it is not the real temperature since the problem is athermal. θ also has the meaning of a Lagrange multiplier for the constraint imposed on S during minimization of U.

If the problem is single-component, as would be the case if the struts in the foam structure are all hinged together so that there is only axial force f but no bending moment or shear force in each strut, then the functional U and S can be expressed in terms of the axial force f and the probability density function $P(f)$ of f. If shear forces (s) and bending moments (m) are present, an elementary analysis shows that U and S can still be expressed as functionals of f, s and m, and their corresponding probability density functions. The variational problem in (1) can therefore be solved analytically to obtain the probability density functions of f, s and m. It can be shown that for the simple hydrostatic and pure shear stress states, these probability density functions are Gaussian if the struts are linear elastic. The model curves in fig. 2(a,b) are such Gaussian curves and the parameter κ shown there is an inverse measure of the temperature analog θ.

For load mixities other than the hydrostatic or pure shear states, the force distributions can be obtained by superposition of the hydrostatic and pure shear states. The result is that the force distributions under general load mixities are given as convolution integrals of Gaussian functions. Fig. 3 shows the predicted axial force distribution (the solid curve) in a 2-D grid under uniaxial loading, which is the convolution of two Gaussian functions centred at 0.5 and 1.5 of the abscissa. There is no fitting parameter involved in the predicted force distribution here and the agreement with the finite element results (the discrete points), especially the capturing of the m-shaped bifurcation behaviour, is excellent. With the force distributions at different load mixities worked out this way, the failure criteria for the structure can be constructed using a survival probability concept. Fig. 4 shows the predicted first-yield and buckling loci (the triangles and squares respectively), in comparison with a plastic collapse mean-field theory¶ available in the literature (the circles).

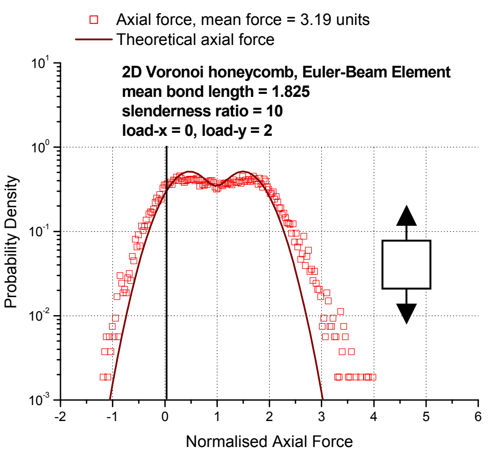

Fig. 3 Axial force distributions of 2-D perturbed square grid under unaxial loading.

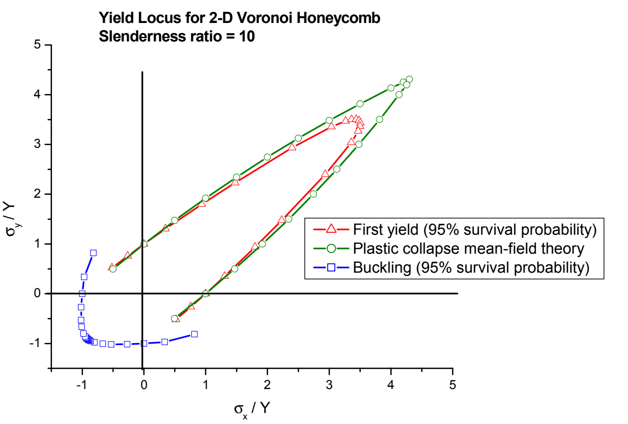

Fig. 4 Predicted yield locus of 2-D perturbed square grid. Y is material yield strength.

¶ Gibson, L.J. and Ashby, M.F., 1988, *Cellular Solids – Structure and Properties*, Pergamon Press (Oxford, UK), p. 103.