PROPAGATION OF CRACKS IN TERMS OF CONTINUUM DAMAGE MECHANICS

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Summary The paper deals with numerical analysis of crack nucleation and propagation caused by time-dependent process of deterioration in creep conditions. Three distinctive stages of the damage growth in a rectangular plate subjected to uniform pressure load are considered: nucleation in a point, propagation through the plate thickness, and development of critical network of the cracks bringing a structure to the final collapse. With corresponding times denoted by t_1, t_2 , and t_3, their relationships had been evaluated to indicate the safety margins throughout the whole process. Spatial configuration of crack networks, including their profiles and branching, is shown as time-dependent process, which leads to the structure collapse caused by the loss of kinematical stability.

INTRODUCTION

Process of failure of structures caused by crack development is very complex one, but in general, three characteristic stages can be distinguished corresponding to the nucleation of first macro-crack, its propagation throughout structure's body, and to the formation of collapse mechanism. The corresponding times limiting these stages are denoted as t_1, t_2 , and t_3. The ratio of the values of these time instances can be viewed as safety margins set on structures' behaviour.
In the paper Continuum Damage Mechanics is used for analysis of damage growth in a metallic rectangular plate subjected to uniform pressure load. Some comparison between approach of Continuum Damage Mechanics and Fracture Mechanics to the problem of failure of structures solving is made.
The set of constitutive equations was chosen as simple as possible to facilitate numerical analysis, but yet complex enough – to reflect typical process of creep of metallic structures (non-stationary creep with steady-state and non-steady periods, stress redistribution, nonlinear dependency of strain rate on stress and coupling between deformation and deterioration):

$$\varepsilon_{ij}^{e} = D_{ijkl}^{-1}\,\sigma_{kl} \tag{1}$$

$$\frac{\partial \varepsilon_{ij}^{c}}{\partial t} = \gamma \left(\frac{\sigma_{eff}}{1-\omega}\right)^{n} \frac{\partial \sigma_{eff}}{\partial \sigma_{ij}} \tag{2}$$

$$\frac{\partial \omega}{\partial t} = A\left(\frac{\sigma_{eq}}{1-\omega}\right)^{m} \tag{3}$$

where: ε_{ij}^{e} - the elastic strain tensor, ε_{ij}^{c} - the creep strain tensor, σ_{ij} - the stress tensor, D_{ijkl} - the elastic constants tensor, ω - the scalar damage parameter, γ, n, A, m - creep and damage material constants, t - time.
The equivalent stress σ_{eq} in Eq. (3) is given by:

$$\sigma_{eq} = \alpha\,\sigma_{\max} + (1-\alpha)\sigma_{eff} \tag{4}$$

where: $\sigma_{\max}$ - the maximal principal tensile stress, σ_{eff} - the Huber – Mises effective stress, α - parameter $(0 \le \alpha \le 1)$ which characterises local failure mechanism mode.
The case of $\alpha = 0$ corresponds to ductile (transgranular) fracture controlled by the effective stress whereas for $\alpha = 1$ the brittle (intergranular) fracture governed by the maximal principal tensile stress occurs. The intermediate values of α correspond to mixed modes of failure.
The above constitutive equations completed together with equilibrium and compatibility equations form the set of problem governing equations that allow for effective solving of a problem of description the whole process of cracking structures.

NUMERICAL PROCEDURES

The set of problem governing equation has been integrated by means of numerical methods. The Finite Element Method for structure discretisation and Euler's procedure for time integration was used. In the computer code the layered isoparametric eight-node Serendipity shell elements with reduced integration were employed. Ten layers and two-point Gaussian quadrature for volume integration were adopted. The time t_1 is identified with $\omega = 1$ condition fulfilled in any layer and Gaussian point (that is numerical integration point). For time $t > t_1$ these numerical integration points were excluded from further integration. When critical condition for damage parameter is reached in all ten layers of a Gaussian points the time is referred to as t_2. Time of structure collapse t_3 were identified with critical values of damage parameter in a whole finite element, which in turn lead to instability in numerical calculations.

SOME NUMERICAL RESULTS

In Fig. 1 upper and lower surfaces of one of the analysed plate (clamped plate, $\alpha = 1$) are shown at time t_2. The networks of macrocracks is shown also, with indication of their onset marked by ○, and through-body proliferation at time t_2 marked by ◄. Though these cracks are seen as surface ones, in fact they penetrate the body of a structure. Profiles of the cracks along two cross sections (along clamped edge and through a mid-span of the plate) are shown in Fig.1 as well.

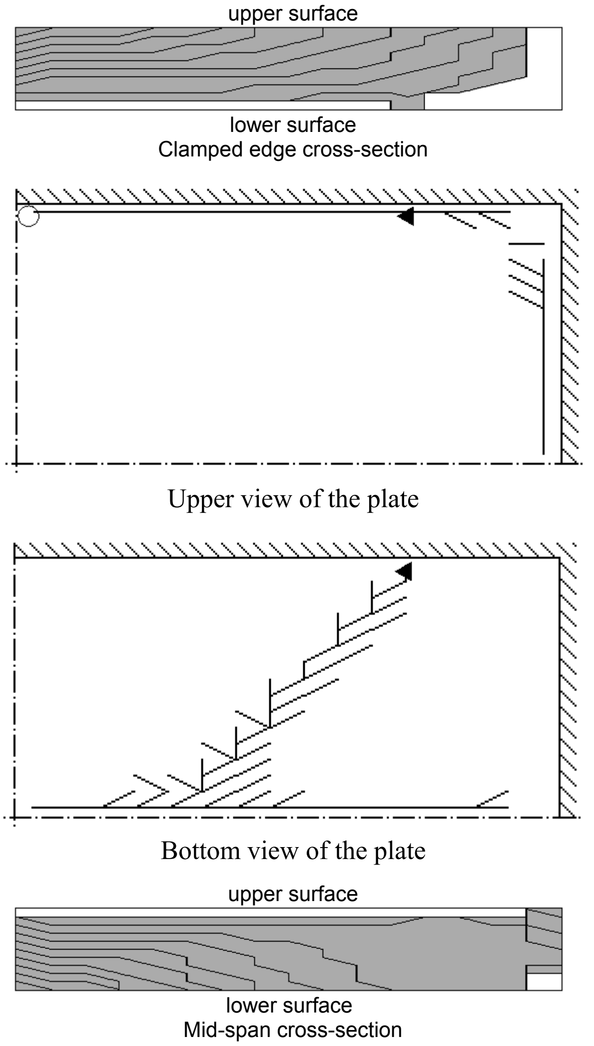

Figure 1: Cracking patterns at time t_2.

An advantage of Continuum Damage Mechanics' approach, fully exploited in the present analysis, is twofold: first, the location for a point at which a macrocrack initiates comes out as a result of analysis. No assumption of this location has to be made as unavoidable assumption prior to further analysis, which is a case when Fracture Mechanics is to be applied. Further, the direction of a macrocracks and their branching it comes out as the result of Continuum Damage Mechanics analysis. Finally, three-dimensional profiles of the cracks penetrating structure's body can be determined.