SIMULATION OF TRACK BALLAST

Dmitry Agapov*, Dmitry Pogorelov*, Aleksandr Bidulya**

* Bryansk State Technical University, Bulvar 50-letiya Oktyabrya 7, 241035 Bryansk, Russia
** All-Russian Research and Design Institute, ul. Oktyabrskoy Revolutsii 410, 140402 Kolomna, Russia

Summary Algorithms for simulation railway track ballast are considered. Mathematical model of ballast, collision detection algorithms and features of numerical methods are discussed. Some simulation results are presented.

INTRODUCTION

There are two principal approaches for simulation of granular system dynamics. The first one represents the system as a continuum media. In the second approach, such systems are considered as a set of interacting rigid bodies (particles). The last method is discussed in the paper. Its main lack is increased requirement to computational resources. However, the first method does not allow accounting the real geometry of interacting particles. Therefore there is the need for improving numerical methods for solving equations of motion of system with great number of degrees of freedom.

MATHEMATICAL MODEL DESCRIPTION

All bodies of the system are considered as planar both convex and non-convex polygons. Each of the particles has three degrees of freedom corresponding to plane motion. A contact interaction between each pair of bodies is implemented by contact forces. They appear when the shapes of bodies overlap and disappear when the shapes are going away from each other.

Contact force model

We consider the contact between two polygons. Since the interception of the polygons is small, we suppose that the vertices of the first polygon are penetrating through the edges of the second one and vice versa. That is why we discuss a model of the vertex-edge contact as a special case of the point-plane contact described in [1]. When the vertex penetrates into the inner side of the edge, the contact forces appear. In a simplest case the contact force consists of a linear viscous-elastic normal force and a dry friction force. The dry friction has two modes: sliding and sticking ones.

Collision detection

The collision detection consists of two levels. The first one is so-called far collision level, which checks interlacing simple hulls of particle shapes. The linked linear list method is used in this work [2]. All the bodies are surrounded with square hulls and ordered lists of the hull bounds for each axis are created. All the pairs of the bodies having far contact are placed to a collision heap. Additional use of circle hulls could reduce the collision heap length. Then the second level of collision detection is carried out for the pairs from the collision heap. The sensitivity cell method is used for this level. The sensitivity cell for an edge of a polygon is a region bounded with the same edge, straight line parallel to the edge and shifted to a distance $d$ from the edge inside the polygon and two inner bisectors of the neighbor angles. The parameter $d$ is called the depth of sensitivity. A vertex of another polygon lies inside the sensitivity cell if several conditions are satisfied. These conditions require calculation of oriented distances form the vertex to the straight lines bounding the sensitivity cell (Figure 1). The oriented distances are calculated as

\[
\begin{align*}
    d_y^g &= (\hat{e}_j \times (\hat{r}_e - \hat{r}_v))_z; \\
    d_y^b &= (\hat{b}_j \times (\hat{r}_b - \hat{r}_v))_z; \\
    d_d &= -(\hat{b}_{j+1} \times (\hat{r}_e - \hat{r}_{v_{j+1}}))_z,
\end{align*}
\]

here $\hat{e}_j$ is the unit vector along $j$-th edge, $\hat{b}_j, \hat{b}_{j+1}$ are the bisectors of $j$-th and $(j+1)$-th angles, $\hat{r}_e, \hat{r}_{j+1}$ are the radius-vectors of $j$-th and $(j+1)$-th vertices, $\hat{r}_v$ is the radius-vector of point $v$. If the vertex of the body is in the polygon of other body, the corresponding contact force appears. Thus the collision detection algorithm is as follows: 1) the loop over all the bodies is preformed with creating linked linear lists for the neighbor collision level; 2) far collision level is checked; 3) neighbor collision level is performed for the pairs of the bodies, which satisfy the far collision level. If the pair of bodies passes the neighbor collision level test, the corresponding contact force element is added; 4) the contact force element is destroyed if the contact is broken; 5) integration step is executed; 6) if simulation is not over go to item 2.
Simplification of the Jacobian matrices of contact forces
The implicit Park’s method with calculation of simplified Jacobian matrices is used for numerical integration of equation of motion [4], [5]. However, the use of this method increases the width of the system matrix profile and decelerates the simulation process in the case of ballast model consisting of thousands of bodies [6]. Therefore, a method of block-diagonal Jacobian matrices was developed. To clarify the method, consider force \( F_j^{ij}(q_i, q_j, \dot{q}_i, \dot{q}_j) \), which acts on the \( i \)-th body. The Jacobian matrix includes derivatives \( \partial F_j^{ij}/\partial q_i \), \( \partial F_j^{ij}/\partial \dot{q}_i \) and does not include \( \partial F_j^{ij}/\partial q_j \), \( \partial F_j^{ij}/\partial \dot{q}_j \). Single integration step expenditure is about 10-20 percents as large as that without Jacobian matrix. However the integration step increases and the computational expenses are reduced. The convergence of this method is improved as well.

REALIZATION AND IMPLEMENTATION
The methods described above were realized for railway track ballast simulation as a module of the program package Universal Mechanism (www.umlab.ru). This module has ability of input the statistics of particle geometry, two methods for filling a work area by particles, horizontal, vertical and combined types of vibrocompactions in the rigid box (work area), compaction by sleepers. The output data is global and local porosity, kinematics of the system and each particle and so on. There exists ability to colouring the particles depending on their velocity or contact power factor (Figure 2). The system of thousand particles is presented in this figure. All the bodies are convex polygons. The amplitude of the vertical vibration is 7 mm and the frequency is 45 Hz.

CONCLUSIONS
The algorithms described in this paper allow simulating dynamics of two-dimensional ballast models including up to several thousands bodies of arbitrary shape. The realized module solves such tasks as ballast packing and compaction. It is supposed to generalize the methods to the three-dimensional case.

ACKNOWLEDGEMENTS
The research was supported by Russian Foundation for Basic Research under the grant 02-01-00364-A as well as by the scientific program “Universities of Russia – Basic Research” (UR.04.01.046).

References