INVERSE PROBLEMS OF THERMOELASTICITY FOR FRICTIONALLY INTERACTING LAYERS

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Summary New inverse thermoelasticity problems for frictionally interacting layers have been formulated, in which unknown thermal loading (temperature of boundary surface, intensity of frictional heat flux) is determined using additionally given vertical displacements of one of the outer boundary surfaces. The functional spaces, in which the problems are well-posed, have been found. The method for solving the problems has been suggested and numerically verified with the use of the solution of the direct problem.

INTRODUCTION

The thermal stressed state of frictionally interacting bodies under known initial-boundary thermal and mechanical conditions can be investigated with the use of well-known methods, such as, for example [1-3]. However, for the working friction couple, the information on thermal loading is often available only on the part of a surface as a result of the restriction of access possibility that makes an appropriate thermoelasticity problem ill-posed. To determine thermal regime and thermal stressed state of the friction couple in this case, it is possible to use additional information and to reduce the initial thermoelasticity problem to the inverse one [4, 5].
The problem of determining the friction coefficient of interacting bodies is also important. The friction coefficient is often considered in calculations as a constant value. In reality the friction coefficient changes in time and depends on many factors, such as normal loading, temperature of bodies, sliding velocity, etc., which are also changeable under the conditions of the non-stationary character of the process. It is shown in the paper that the problem of finding the unknown law of change in time of the layers friction coefficient under known boundary and initial conditions can be solved by using additionally measured vertical displacements of one of the outer boundary surfaces of the friction couple, that is by reducing it to the solution of the inverse problem.
Using the found change in time of the friction coefficient and known sliding velocity and contact pressure, it is possible to determine the change in time of the intensity of the frictional heat flux, which is important for adequate calculation of the friction couple and for the choice of rational regimes of its work.

STATEMENT OF PROBLEMS

Let us consider a one-dimensional model of the friction contact of two different layers. We assume that the lower surface of the first layer is elastically fixed, while the second layer is moving with variable speed over the first one and is pressed to it. As a result of friction on the contact surface, heat is formed. The intensity of the total frictional heat flux equals the specific power of friction forces. The mechanical contact of layers is assumed to be ideal, and the thermal contact is considered as non-ideal.
Within this assumed framework, the following problems are investigated:

- problem 1 consists in determining unknown thermal loading on the fixed surface of the friction couple by given temperature and vertical displacements of the other outer boundary surface;
- in problem 2, the change in time of the layers friction coefficient (intensity of frictional heat flux) is determined by additionally known vertical displacements of the outer loaded boundary surface of the friction couple under given initial and boundary thermal conditions.

The formulated problems are inverse thermoelasticity problems, in which the cause – thermal loading (temperature of the boundary surface) in problem 1 and the intensity of the frictional heat flux (friction coefficient) in problem 2 – is determined by the displacements of the boundary surface.

SOLUTION OF PROBLEMS

Using the well-known methods of solving equations of the appropriate contact problem of thermoelasticity for the layers under consideration [3], we represent vertical displacements of the second layer in the form of functional dependence upon the distribution of temperature of each layer. If we substitute into this dependence the temperatures of layers found with the help of the Laplace integral transformation in the form of functional dependences on loading thermal factors and take into account that the displacements of the outer boundary surface of the second layer are known, we will obtain the convolution type Volterra integral equations of the first kind

$$\int_0^t K_i(t-x)Q_i(x)dx = F_i(t), \qquad 0 \le x \le t, \qquad i = 1,2 \tag{1}$$

for determining at the time moment $t \in [0, t_*]$ the unknown function $Q_1(t)$ – the temperature of the boundary surface in problem 1 and $Q_2(t)$ – the intensity of the frictional heat flux (friction coefficient) in problem 2. In equations (1) F_i are known functions represented through the prescribed displacements and functions, which are given in the boundary and initial conditions, $K_i(t-x)$ are known kernels.

The investigation of the kernel $K_1(t-x)$ has shown that it is possible to find the unique continuous stable solution $Q_1(t) \in C[0, t_*]$ of problem 1 using the Laplace integral transformation, if input functions (displacements, temperature of the surface, intensity of the frictional heat flux) are twice continuously differentiable from space $C^2[0, t_*]$. This means that in the indicated functional spaces problem 1 is well-posed. Integral equation (1), which corresponds to problem 2, can be reduced to the Volterra integral equation of the second kind with the kernel with integrable singularity. The unique continuous stable solution $Q_2(t) \in C[0, t_*]$ of the obtained equation can be found by means of the method of averaged functional corrections [6]. Here the inverse problem is well-posed if input functions (displacements, temperatures of surfaces) are continuously differentiable from space $C^1[0, t_*]$.

The obtained solutions of the formulated problems allow us to investigate the change in time of functions $Q_i(t)$ during the whole period of bodies interaction as well as dependence of the friction coefficient on the determining parameters of the process (such as sliding velocity, contact pressure, temperature of contact surface).

NUMERICAL VERIFICATION

For the friction couple bronze-steel, the numerical verification of the suggested method for solving the formulated problems has been carried out. With this purpose, we prescribed boundary thermal conditions and determined the change in time of vertical displacements of the outer boundary surface of the second layer as a solution of the appropriate direct contact thermoelasticity problem. The found displacements were approximated by cubic spline with accuracy $d = 0.1\%$ and were used as given in inverse problems. Using the suggested here approach, we determined the solutions of inverse problems – temperature of the boundary surface (problem 1) and the friction coefficient (problem 2). Comparison of the found solutions with the functions given in direct problems has shown that maximal relative deviation of the solutions from appropriate known functions does not exceed $e = 0.8\%$ in problem 1 and $e = 1.4\%$ in problem 2 that provides evidence of satisfactory accuracy of obtained inverse problems solutions. It should be noted that the decrease of the error of displacements approximation d leads to the decrease of relative deviation e that numerically confirms stability of obtained solutions.

CONCLUSIONS

1. Within the framework of the one-dimensional model of frictional interaction of different layers, new inverse thermoelasticity problems have been formulated, in which unknown thermal loading (temperature of the outer boundary surface, frictional heat flux) is determined by the additionally known change in time of vertical displacements of one of the outer boundary surfaces.
2. The method for solving formulated problems has been suggested based on their reduction to the Volterra integral equations with further use of the Laplace integral transformation and of the method of averaged functional corrections.
3. On the basis of the analysis of the formulated problems, the functional spaces in which the problems are well-posed have been determined.
4. For the friction couple bronze-steel, the numerical verification of the suggested method for solving the formulated problems has been performed, which confirmed its efficiency.

References

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