AN APPROACH TO WORM-LIKE MOTION

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Summary A worm-like motion system in form of material points, connected by a spring is considered. It is shown that at presence of internal excitation and non-symmetric Coulomb dry friction a motion of the system with a constant "on the average" velocity is possible and this motion is stable. The expression for the velocity is obtained. A worm prototype applying the principles outlined above has been constructed.

MATHEMATICAL MODEL

Observing the locomotion of worms one recognizes a conversion of (mostly periodic) internally driven motions into change of external position (undulatory locomotion). In [1] the motion of three material points in a common straight line with the method of direct separation of motion [2] is considered. Left and right points are equipped with scales contacting the ground, and middle point is under action of harmonic external force. In this paper the motion of a system of two material points x_1 and x_2 with the masses m, connected by a spring of stiffness c along an axis x is considered (Fig.1).

It is supposed that the points are under the action of a small non-symmetric Coulomb dry frictional force $\varepsilon\, m F(\dot{x})$, $\varepsilon << 1$, depending on velocities $\dot{x} = \dot{x}_i \,(i = 1,2)$, where $F(\dot{x}) = F_+$ if $\dot{x} > 0$, $F(\dot{x}) = -F_-$ if $\dot{x} < 0$, $F(\dot{x}) = F_0$ if $\dot{x} = 0$; $-F_- < F_0 < F_+$, $F_- \geq F_+ \geq 0$. Excitation is carried out due to action of small internal forces $G(t) = \varepsilon\, m(b_0 + b\cos\psi)$, $\psi = \nu t$. Such forces arise, for example, if the spring is a magnetizable elastic material by influence of an external magnetic field [2]. The equations of the motion are:

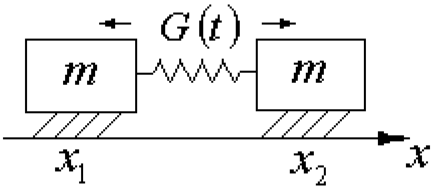

Fig.1. Model of the system

$$
\begin{aligned}
&m\ddot{x}_1 + c(x_1 - x_2 + l_0) + \varepsilon\, m(b_0 + b\cos\psi) + \varepsilon\, mF(\dot{x}_1) = 0,\\
&m\ddot{x}_2 + c(x_2 - x_1 - l_0) - \varepsilon\, m(b_0 + b\cos\psi) + \varepsilon\, mF(\dot{x}_2) = 0.
\end{aligned} \tag{1}
$$

The system (1) is integrable on intervals where $F(\dot{x})$ is constant. However such approach does not give an opportunity of a qualitative description of motion as whole because the solutions "stick together" on intervals. Designating $\omega^2 = c/m$, and replacing $x_2 \to x_2 - l_0 - \varepsilon\, b_0/\omega^2$ we receive a system of equations, holding the old symbols:

$$
\begin{aligned}
&\ddot{x}_1 + \omega^2 (x_1 - x_2) = -\varepsilon\,[F(\dot{x}_1) + b\cos\psi],\\
&\ddot{x}_2 + \omega^2 (x_2 - x_1) = -\varepsilon\,[F(\dot{x}_2) - b\cos\psi].
\end{aligned} \tag{2}
$$

To system (2) we apply procedure of averaging according to [4]. For this purpose we introduce variables: the velocity of the center of mass $V = (\dot{x}_1 + \dot{x}_2)/2$ and a deviation of points relatively to the center of mass $z = (x_2 - x_1)/2$. Non-disturbed system ($\varepsilon = 0$) has two integrals: $V = const$ and the amplitude of oscillations relatively to the center of mass a is constant, $z = a\cos\varphi$, $\varphi = \Omega t + \vartheta$, $\Omega = \sqrt{2}\,\omega$, ϑ - is an arbitrary constant. Replacing $V = (\dot{x}_1 + \dot{x}_2)/2$, $z = a\cos(\Omega t + \vartheta)$, $\dot{z} = -a\Omega\sin(\Omega t + \vartheta)$, where V, a, ϑ - slow variables, we receive system(2) in a standard form[4]:

$$
\begin{aligned}
&\dot{V} = -\frac{\varepsilon}{2}\left[F(V + a\Omega\sin\varphi) + F(V - a\Omega\sin\varphi)\right],\\
&\dot{a} = -\frac{\varepsilon}{2\Omega}\sin\varphi\,\left[F(V + a\Omega\sin\varphi) - F(V - a\Omega\sin\varphi) + 2b\cos\psi\right],\\
&\dot{\varphi} = \Omega - \frac{\varepsilon}{2a\Omega}\cos\varphi\,\left[F(V + a\Omega\sin\varphi) - F(V - a\Omega\sin\varphi) + 2b\cos\psi\right],\\
&\dot{\psi} = \nu\,.
\end{aligned} \tag{3}
$$

We investigate the system (3) in a vicinity of the main resonance $\nu = \Omega + \varepsilon\Delta$, $\Delta \neq 0$. For this purpose we introduce a new slow variable $\xi = \psi - \varphi$ and we exclude a fast variable ψ. After averaging on a fast variable φ we obtain:

$$\dot{V} = \begin{cases} -\varepsilon\left(\dfrac{F_- + F_+}{\pi}\arcsin\dfrac{V}{a\Omega} - \dfrac{F_- - F_+}{2}\right) & \text{if } 0 \le V < a\Omega, \\ -\varepsilon F_+ & \text{if } V \ge a\Omega, \end{cases}$$
$$\dot{a} = \begin{cases} -\dfrac{\varepsilon}{\Omega}\left(\dfrac{F_- + F_+}{\pi}\sqrt{1 - \dfrac{V^2}{a^2\Omega^2}} - \dfrac{b}{2}\sin\xi\right) & \text{if } 0 \le V < a\Omega, \\ \varepsilon\dfrac{b}{2\Omega}\sin\xi & \text{if } V \ge a\Omega, \end{cases} \qquad (4)$$
$$\dot{\xi} = \varepsilon\left(\frac{b}{2a\Omega}\cos\xi + \Delta\right).$$

We are interested in an approximately steady motion as a single whole, therefore we seek for the solution $\dot{V} = 0$. Then from (4) we have $\dot{a} = \dot{\xi} = 0$ and

$$V = \frac{\sin\Phi}{|\Delta|}\sqrt{\frac{b^2}{4} - \frac{(F_- + F_+)^2\cos^2\Phi}{\pi^2}}, \qquad a = \frac{V}{\Omega\cdot\sin\Phi},$$
$$\xi = \arccos\left[-\frac{\Delta}{|\Delta|}\sqrt{1 - \frac{4(F_- + F_+)^2\cos^2\Phi}{\pi^2 b^2}}\right], \qquad \Phi = \frac{\pi}{2}\cdot\frac{F_- - F_+}{F_- + F_+}. \qquad (5)$$

The result $V(t)$ of the numerical integration of the exact system (1) is given in Fig.2. The following values of parameters were taken: $\varepsilon = 0.01$; $F_+ = 1$; $F_- = 2$; $b = 10$; $\Delta = 10$, $\omega = 1$. The formula (5) gives the value for the velocity of center of mass $V = 0.24$. Steady motion with the constant velocity (5) is physically feasible only in the event that it is stable. The characteristic polynomial of system in the variations, received of system (4), is given by expression (6). According to the Hurwitz criterion, all roots of the characteristic polynomial (6) have the negative real part for all values of parameters.

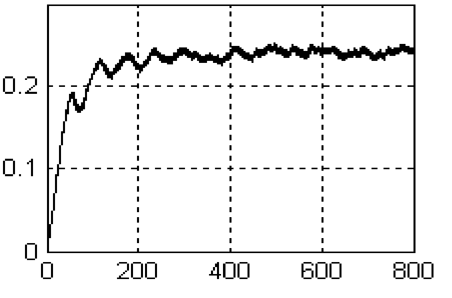

Fig.2. Velocity vs. time

$$P(\lambda) = \lambda^3 + \lambda^2\cdot\varepsilon\frac{2(F_- + F_+)}{\pi a\Omega}\cdot\frac{1}{\cos\Phi} + \lambda\cdot\varepsilon^2\left[\frac{(F_- + F_+)^2}{\pi^2 a^2\Omega^2}(1 + \sin^2\Phi) + \Delta^2\right] + \varepsilon^3\frac{(F_- + F_+)\Delta^2}{\pi a\Omega\cos\Phi}. \qquad (6)$$

So, the motion with a velocity V is stable.

CONCLUSIONS

The above fulfilled investigations show: at presence of excitation and non-symmetrical Coulomb dry friction, motion of system with a constant "on the average" velocity $V > 0$ is possible and this motion is stable.
Worm prototype applying the principles outlined above has been constructed and proved positive.

References

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