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NUMERICAL COMPUTATION OF COMPRESSIBLE VISCOUS FLOW THROUGH A MALE ROTOR-HOUSING GAP OF SCREW COMPRESSORS

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Summary This contribution deals with the numerical computation of a compressible viscous fluid flow through a two-dimensional model of the male rotor-housing gap in the screw compressor. Numerical solution of the nonlinear conservative system of the compressible Navier-Stokes equations is obtained by means of the cell-centred finite volume formulation of the explicit two-step TVD MacCormack scheme proposed by Causon on a structured quadrilateral grid using the own numerical code.

INTRODUCTION

Gas leakage is a phenomenon that has a lot of different features, many of significant importance. Compressor engineers are mostly interested in estimation for the mass flow rate. It has a great influence on the compressor performance, especially with regard to its internal efficiency. Therefore it is necessary to make reasonable estimates for mass flow rates or to investigate the details of the leakage flow. The aim of this contribution is to show the numerical computation of a compressible viscous fluid flow through the male rotor-housing gap (the gap between the stator and the head of the male rotor tooth), Fig. 1, in the screw compressor for the pressure ratio $p_{01}/p_2 = 2$. It is assumed that the leakage flow is laminar in most cases and that the male rotor-housing gap can be simulated by a two-dimensional bounded domain $\Omega \subset \mathbb{R}^2$, occupied by the perfect gas, with the boundary $\partial\Omega = \partial\Omega_I \cup \partial\Omega_O \cup \partial\Omega_W$, where $\partial\Omega_I$ is the inlet and $\partial\Omega_O$ the outlet section of the computational domain Ω, $\partial\Omega_W = \partial\Omega_{WR} \cup \partial\Omega_{WS}$ are impermeable walls of the computational domain corresponding to the stator $\partial\Omega_{WS}$ and the head of the male rotor tooth $\partial\Omega_{WR}$.

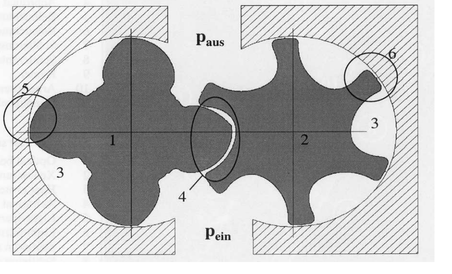

Fig. 1. Frontal section of rotors, 5 – male rotor-housing gap

MATHEMATICAL MODEL OF LAMINAR COMPRESSIBLE FLOW

Let $(0, \mathcal{T})$ be a time interval. The mathematical model of a laminar compressible flow is described by the nonlinear conservative system of the Navier-Stokes (NS) equations. For 2-D problems it can be written in nondimensional form as

$$\frac{\partial \boldsymbol{w}}{\partial t} + \frac{\partial \boldsymbol{f}(\boldsymbol{w})}{\partial x} + \frac{\partial \boldsymbol{g}(\boldsymbol{w})}{\partial y} = \frac{1}{Re_\infty}\left[\frac{\partial \boldsymbol{f}_V(\boldsymbol{w})}{\partial x} + \frac{\partial \boldsymbol{g}_V(\boldsymbol{w})}{\partial y}\right] \qquad \text{in } \Omega \times (0, \mathcal{T}). \tag{1}$$

The vector $\boldsymbol{w}$ of conservative variables and the inviscid $\boldsymbol{f}(\boldsymbol{w}), \boldsymbol{g}(\boldsymbol{w})$ and viscous fluxes $\boldsymbol{f}_V(\boldsymbol{w}), \boldsymbol{g}_V(\boldsymbol{w})$ are defined as

$$\boldsymbol{w} = \big(\varrho,\, \varrho u,\, \varrho v,\, E\big)^T, \quad \boldsymbol{f}(\boldsymbol{w}) = \big(\varrho u,\, \varrho u^2 + p,\, \varrho uv,\, (E+p)\,u\big)^T, \quad \boldsymbol{g}(\boldsymbol{w}) = \big(\varrho v,\, \varrho uv,\, \varrho v^2 + p,\, (E+p)\,v\big)^T, \tag{2}$$

$$\boldsymbol{f}_V(\boldsymbol{w}) = (0,\, \tau_{xx},\, \tau_{xy},\, u\tau_{xx} + v\tau_{xy} - q_x)^T, \quad \boldsymbol{g}_V(\boldsymbol{w}) = (0,\, \tau_{yx},\, \tau_{yy},\, u\tau_{yx} + v\tau_{yy} - q_y)^T, \tag{3}$$

where t is time, ϱ density, p static pressure, E total energy per unit volume, u, v are Cartesian components of velocity vector $\boldsymbol{v}$ and $\tau_{xx}, \tau_{xy}, \tau_{yx}, \tau_{yy}$ are laminar shear stresses given for a Newtonian fluid. The external volume forces are neglected.

To close the system of the compressible NS equations (1) it is necessary to specify the equation of state for the perfect gas $p = \varrho\, r\, T = (\kappa - 1)\, \varrho\, c_v\, T$ and the relation for total energy per unit volume $E = \frac{p}{\kappa - 1} + \frac{1}{2}\varrho|\boldsymbol{v}|^2$, where T is thermodynamic temperature, $r = c_p - c_v$ the gas constant per unit mass, c_p and c_v are the specific heats at constant pressure and volume, respectively and $\kappa = 1.4$ is so-called Poisson's constant. Heat flux terms q_x, q_y can be written as

$$q_x = -k\frac{\partial T}{\partial x} \equiv -\frac{\kappa}{\kappa - 1}\,\frac{\eta}{Pr}\,\frac{\partial}{\partial x}\left(\frac{p}{\varrho}\right), \qquad q_y = -k\frac{\partial T}{\partial y} \equiv -\frac{\kappa}{\kappa - 1}\,\frac{\eta}{Pr}\,\frac{\partial}{\partial y}\left(\frac{p}{\varrho}\right), \tag{4}$$

where η is molecular viscosity, k thermal conductivity and $Pr = c_p\eta/k$ is the laminar Prandtl number which is taken to be 0.72 for the calorically perfect gas. All quantities are considered as nondimensional. The reference Reynolds number is defined as $Re_\infty = \varrho_{ref} u_{ref} l_{ref}/\eta_{ref}$.

Boundary conditions

We consider the flow with $Re_\infty = 3900$. At the inlet $\partial\Omega_I$, the stagnation pressure $p_{01} = 1$, the stagnation temperature $T_{01} = 1$, the inlet angle α_1, $\frac{\partial T}{\partial n} = 0$ and $\tau^{ij} n_j = 0$, $i = 1, 2$ are prescribed. At the outlet $\partial\Omega_O$, the static pressure $p_2 = 0.5$, $\frac{\partial T}{\partial n} = 0$ and $\tau^{ij} n_j = 0$, $i = 1, 2$ are kept. On the walls $\partial\Omega_{WS}$ and $\partial\Omega_{WR}$, the boundary conditions $u = 0$, $v = 0$ and $\frac{\partial T}{\partial n} = 0$ are satisfied. It is assumed that the male rotor does not move. These nondimensional boundary conditions give the inlet Mach number $M_1 = 0.05$ and the outlet Mach number $M_2 = 0.63$, see Fig. 2 and Fig. 3.

NUMERICAL METHOD

For the discretization of the nonlinear conservative system of the compressible NS equations in nondimensional form (1) the cell-centred finite volume (FV) method on a structured quadrilateral grid is used. Time integration of the inviscid part of the system (1) is performed by using the FV formulation of the explicit two-step Causon's simplified TVD MacCormack scheme, cf. [1, 2] for its derivation. The approximations of the viscous part of the system (1) are added to the predictor and corrector steps of the MacCormack scheme

$$\boldsymbol{w}_{ij}^{n+\frac{1}{2}} = \boldsymbol{w}_{ij}^{n} - \frac{\Delta t}{|\Omega_{ij}|} \sum_{m=1}^{4} \left(\boldsymbol{f}_m^n S_m^x + \boldsymbol{g}_m^n S_m^y \right) + \frac{\Delta t}{|\Omega_{ij}|} Visc(\boldsymbol{w}_{ij}^n) \,, \tag{5}$$

$$\overline{\boldsymbol{w}_{ij}^{n+1}} = \frac{1}{2} \left\{ \boldsymbol{w}_{ij}^{n} + \boldsymbol{w}_{ij}^{n+\frac{1}{2}} - \frac{\Delta t}{|\Omega_{ij}|} \sum_{m=1}^{4} \left(\boldsymbol{f}_m^{n+\frac{1}{2}} S_m^x + \boldsymbol{g}_m^{n+\frac{1}{2}} S_m^y \right) \right\} + \frac{1}{2} \frac{\Delta t}{|\Omega_{ij}|} Visc(\boldsymbol{w}_{ij}^{n+\frac{1}{2}}) \,, \tag{6}$$

$$^{(TVD)}\boldsymbol{w}_{ij}^{n+1} = \overline{\boldsymbol{w}_{ij}^{n+1}} + d\boldsymbol{w}_{ij}^{1n} + d\boldsymbol{w}_{ij}^{2n} \,, \tag{7}$$

where $^{(TVD)}\boldsymbol{w}_{ij}^{n+1}$ is the corrected numerical solution at time t_{n+1}, $|\Omega_{ij}|$ denotes the face area of the finite volume Ω_{ij}, the inviscid numerical fluxes $\boldsymbol{f}_m^n$ through the edges Γ_{ij}^m, $m = 1, \dots, 4$, of the cell Ω_{ij} at time t_n are evaluated as $\boldsymbol{f}_1^n = \boldsymbol{f}_{i+1j}^n$, $\boldsymbol{f}_2^n = \boldsymbol{f}_{ij+1}^n$, $\boldsymbol{f}_3^n \equiv \boldsymbol{f}_4^n = \boldsymbol{f}_{ij}^n$ and at time $t_{n+\frac{1}{2}}$ as $\boldsymbol{f}_1^{n+\frac{1}{2}} \equiv \boldsymbol{f}_2^{n+\frac{1}{2}} = \boldsymbol{f}_{ij}^{n+\frac{1}{2}}$, $\boldsymbol{f}_3^{n+\frac{1}{2}} = \boldsymbol{f}_{i-1j}^{n+\frac{1}{2}}$, $\boldsymbol{f}_4^{n+\frac{1}{2}} = \boldsymbol{f}_{ij-1}^{n+\frac{1}{2}}$. The numerical fluxes $\boldsymbol{g}_m^n$ are computed in the same way. $\boldsymbol{S}_m = \left(S_m^x \,,\, S_m^y\right)^T$ are cell side normal vectors to the edges Γ_{ij}^m, where $\boldsymbol{S}_1 = \boldsymbol{S}_{i+\frac{1}{2}j}$, $\boldsymbol{S}_2 = \boldsymbol{S}_{ij+\frac{1}{2}}$, $\boldsymbol{S}_3 = \boldsymbol{S}_{i-\frac{1}{2}j}$ and $\boldsymbol{S}_4 = \boldsymbol{S}_{ij-\frac{1}{2}}$.
The viscous terms $Visc(\boldsymbol{w}_{ij})$ in (5) and (6) are approximated by using a FV version with central differences, cf. [3] for details. The added one-dimensional TVD-type viscosity terms $d\boldsymbol{w}_{ij}^{1n}$ and $d\boldsymbol{w}_{ij}^{2n}$ in the direction of change of index i and j respectively, are proposed by Causon, cf. [1].

NUMERICAL RESULTS

For the laminar flow computation through a two-dimensional model of the male rotor-housing gap in the screw compressor, Fig. 2, the relatively fine computational grid with 250×76 quadrilateral cells is used. In order to resolve the boundary layer with sufficient accuracy, the computational grid is refined in the direction normal to the walls. Fig. 2 displays the Mach number distribution in the male rotor-housing gap after 267000 iterations. See Fig. 3 for detail.

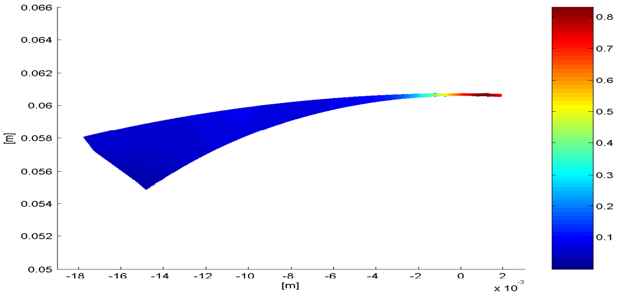

Fig. 2. Mach number distribution

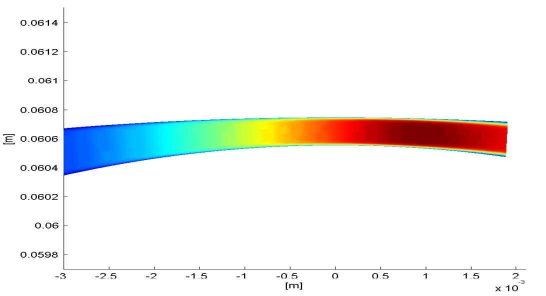

Fig. 3. Detail of the male rotor-housing gap

CONCLUSIONS

The numerical computation of the compressible laminar fluid flow through the male rotor-housing gap of screw compressors is discussed and the obtained results in the form of the Mach number distribution are shown. In the contribution is assumed that the male rotor does not move. At the time of the Congress the numerical results gained for the case of rotary motion of the male rotor with angular velocity $\omega_3 = 523\, rad/s$ will be presented too. In the near future I would like to include the effect of turbulence in a flow field and perform the turbulent computations in the male rotor-housing gap using the algebraic Baldwin-Lomax turbulence model, cf. [4], which can be simply implemented into the own numerical code.
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