THE PROPAGATION OF VISCOUS GRAVITY CURRENTS OVER A RIGID CONIC SURFACE

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Summary Asymptotic models of a thin-film flow of highly viscous heavy fluid with mass supply on a curved rigid surface are constructed. A self-similar solution to the problem of propagation of gravity currents over a conical surface, which can be used for describing extrusive volcano eruptions, is obtained. Non-self-similar flow regimes are investigated numerically.

Formulation of the problem. We consider a non-axisymmetric flow of highly viscous heavy incompressible fluid over a rigid conical surface with local mass supply on the bottom. The problem formulation contains a small parameter ε representing the inverse nondimensional gravity force; accordingly, the solution is sought in the form of asymptotic series in ε. Substituting these series in the governing Navier-Stokes equations and retaining the leading terms, we obtain two fundamentally different systems of equations corresponding to small and finite inclination angles of the surface generatrix to the horizontal. For finite inclinations, the dynamic equation represents the balance between the gravity and the viscous forces, while for small inclination angles the system contains a self-induced longitudinal pressure gradient related with the unknown free-surface shape via the dynamic boundary condition.

Steady-state solutions. The existence of a steady-state solution for the free-surface shape (absent in the case of a horizontal plane) is shown. For small inclination angles, a one-parameter family of solutions on the scale of the mass supply region is obtained for a number of examples. Some of these solutions can be interpreted as a solution of the unsteady problem within the quasi-steady formulation. For finite inclinations and for length scales, much greater than the mass supply region, a unique analytical solution is found. The solutions obtained are generalized for the case of viscoplastic fluid.

Self-similar lava dome growth over a conical surface. We consider an unsteady flow from a point source over a slightly non-axisymmetric conical surface specified by the function $\theta = \theta(\varphi)$ (θ - inclination angle, φ - polar angle in the horizontal plane, $\theta'(\varphi) \ll 1$). For the stretched nondimensional film thickness h, we obtain the hyperbolic partial differential equation (containing φ as a parameter):

$$\frac{\partial h}{\partial t} + \left[\frac{\partial h}{\partial x} + \frac{h}{3x}\right] h^2 \sin\theta(\varphi) = 0 \qquad (1)$$

For a power or an exponential dependence of the entire liquid volume on time, an unique self-similar solution for the free-surface shape and the law of the leading flow front propagation is found analytically. It is shown that Eq. (1) has only two classes of self-similar solutions corresponding to power and exponential laws of mass supply. The families of self-similar solutions describing flows with mass sources or sinks at the leading flow front are also obtained. In the case when the entire fluid volume increases with time as $Qt^{*\gamma}$, the law of flow front propagation in dimensional variables takes the form (dimensional variables are denoted by the asterisk):

$$x_r^* = C(\gamma)\left[\frac{\sin\theta(\varphi)}{\cos^2\theta(\varphi)}\right]^{1/5}\left(\frac{\rho g Q^2}{\mu}\right)^{1/5}(t^*)^{(2\gamma+1)/5}$$

Non-self-similar flow over conical surface with small inclination of the generatrix to the horizontal. For 3D flow over a surface with small inclination angles, a second-order partial differential equation for the free-surface shape is obtained. To compare with the results obtained for finite inclination angles and with the solution for a horizontal plane [1], an axisymmetric flow is studied. It is shown numerically that, in the case of a power law of mass supply, the solution of the evolution equation for h has similarity asymptotics which coincide with the self-similar solution obtained for finite inclination angles. Hence, taking even small inclination angles of the surface generatrix to the horizontal into account leads to the change in the asymptotic law of flow front propagation, as compared with the corresponding law for the horizontal plane.

The solutions obtained can be used for describing extrusive and effusive volcano eruptions on curved substrate surfaces.

References

1. Huppert H.E. The propagation of two-dimensional and axisymmetric viscous gravity currents over a rigid horizontal surface. J. Fluid Mech. 1982. V. 121. P. 43-58.