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MATERIAL INSTABILITIES OF FIBER-REINFORCED NONLINEARLY ELASTIC SOLIDS

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Summary Fiber kinking, fiber splitting, fiber de-bonding and matrix failure are addressed within the context of nonlinear elasticity for fiber-reinforced materials under plane deformation. Fiber failure is given by loss of ellipticity of the governing differential equations.

EXTENDED SUMMARY

A continuum mechanical model designed to predict fiber instabilities in fiber reinforced composite materials is proposed in the context on nonlinear elasticity theory and examined under plane deformation extending the analysis given recently by Merodio and Ogden [1,2]. The (material) instabilities under consideration include fiber kinking, fiber de-bonding, fiber splitting and matrix failure of fiber-reinforced composite materials. In particular, the material models examined are isotropic nonlinearly elastic models augmented with a function that accounts for the existence of a unidirectional reinforcing. This function endows the material with its anisotropic character and is called a *reinforcing model*. The onset of failure is associated with the incipient loss of elliptcity of the governing differential equations (or, equivalently, of the material model). Previous work has dealt with the analysis of specific reinforcing models (see [1] and [2] for references), in particular the so-called *standard reinforcing model*. It was established that the loss of ellipticity for these augmented isotropic materials requires contraction in the reinforcing direction. The loss of ellipticity was related to fiber kinking. Here we generalize these results and establish sufficient conditions to guarantee the ellipticity of the governing equations of equilibrium for more general reinforcing models. Both compressible and incompressible materials are considered. The incipient loss of ellipticity is interpreted in terms of fiber kinking, fiber de-bonding, fiber splitting and matrix failure in fiber-reinforced composite materials.

The loss of ellipticity condition provides the deformation associated with the existence of surfaces of weak discontinuity as well as the directions of the normals to such surfaces for any particular strain-energy function. Surfaces of weak discontinuity (or weak surfaces) are surfaces across which the second derivative of the displacement field is discontinuous (a strong surface of discontinuity, across which the first derivative is discontinuous, is necessarily also a weak surface). In the present analysis, we relate the angle between the weak surface and the fiber reinforcement direction to a particular failure mechanism for a given material model. The onset of fiber kinking is associated with weak surfaces close to the normal of the fiber-reinforcement direction under fiber contraction [3]. Hence, if the loss of ellipticity analysis gives a weak surface perpendicular to the fiber under fiber contraction, the associated fiber failure may be identified as fiber kinking. On the other hand, the angle between the weak surface and the fiber reinforcement is close to zero for fiber debonding [4]. For fiber kinking with fiber splitting, the simultaneous existence of weak surfaces close to and normal to the fiber direction is required [5]. Matrix failure is associated with weak surfaces perpendicular to the fiber reinforcement under fiber extension [6].

In three dimensions, two invariants are sufficient to characterize the anisotropic nature of a transversly isotropic material model. These invariants are deformation measures. One of these two invariants is related only to the fiber stretch of the fiber reinforcement. It is denoted by I_4. The standard reinforcing model is a quadratic function that depends only on this invariant. The other invariant, denoted I_5, is also related to the fiber stretch but introduces an additional effect that influences directly the behavior of the reinforcement under shear deformations. For plane deformation containing the fiber direction these invariants are not independent. Hence, the ellipticity analysis for a general strain-energy function depends on at most one transversly anisotropic invariant. Nevertheless, both invariants are considered since each invariant adds a distinct anisotropic character to the isotropic base material.

First, the ellipticity status of general reinforcing models depending on I_4 is established. The result is the same for both compressible and incompressible materials. It is shown that fiber failure is expected in fiber compression. Under compressive loading in the fiber direction the incipient loss of ellipticity is related to fiber kinking. Fiber failure can also involve fiber extension if the reinforcing model loses convexity, in which case fiber debonding is the expected failure mode. The ellipticity analysis and its relation to fiber failure for reinforcing models depending on the invariant I_5 is also established. In this case the results for compressible and incompressible materials are quite different for fiber extension but similar for fiber contraction. Under compressive loading in the fiber direction, fiber kinking and fiber splitting are the expected failure modes for either compressible or incompressible materials. On the other hand, matrix failure is the expected failure mode under tensile loading in the fiber direction for compressible materials. For incompressible materials, it is shown that the expected failure mode is a weak surface neither perpendicular nor parallel to the fiber direction. This is a failure mode often associated with isotropic materials and corresponds to the initiation of shear bands. Finally, it is shown that convex reinforcing models can fail under fiber extension regardless of the specific form of the model.

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