DYNAMICS OF A ROTOR ROLLING ALONG A CIRCULAR SURFACE

Alla D. Firsova
Institute for Problems in Mechanical Engineering RAS, Department of Vibrational Mechanics, Bolshoy pr. V.O., 61, 199178 St. Petersburg, Russia

Summary This work is devoted to investigating the dynamics of centrifugal-vibrational concentrator (CVC) — recently invented device for using in mining industry to separate particles of granular materials according to their densities. Operating process in CVC is based on using both vibration and centrifugal forces. Experience shows that optimal motion of the CVC is a regular precession of the shaft along an inner surface of the hub without sliding. The aim of the presented work is to determine parameters of such motion and to study its stability as well as to consider transient motions at different types of friction.

DYNAMICAL SCHEME OF THE DEVICE

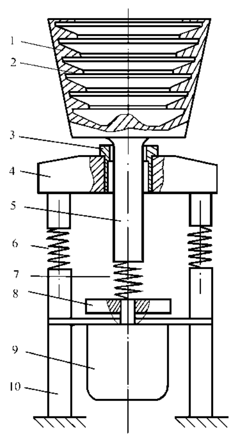

Fig. 1. Dynamical scheme of CVC

Dynamical scheme of CVC is presented in Fig. 1. Here 1 is an operating cup; 2 are fillets, where heavy particles are gathered; 3 is a hub, along which inner surface rolls a shaft 5; 4 is a massive plate; 5 is the shaft rigidly put to the cup 1; 6 are vibration isolators; 7 is elastic element between a shaft of a motor 9 and the shaft 5; 8 is a flyer; 9 is the motor; 10 is a supporting frame.
All parts of the device could be separated into three groups:

- operating cup 1 and details rigidly put to the cup
- plate 4 and details rigidly put to the plate
- frame 10 and details rigidly put to the frame

The details from the first group during operation process rotate together with the shaft of the motor, let us call them “a rotor”. The second group consists of details oscillating during operation process, we will call these details “a stator”. The rest details, “a frame” do not move during operation process.

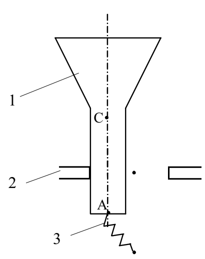

Fig. 2. Considered model.

Let us consider interaction between the rotor and stator according to scheme presented in Fig. 2. Here 1 is the rotor; point C is its centre of mass; 2 is the hub (a detail of stator along which the rotor rolls; 3 is a supporting elastic element; point A is a point, where elastic element is put. We believe that height of a hub surface is negligibly small as compared to rotor size, i.e. even in vertical position the rotor contacts the hub in one point. The second assumption is that mass of the stator is much more that mass of the rotor and when we study motion of the rotor we can take no notice of stator movements.

EQUATIONS OF MOTION

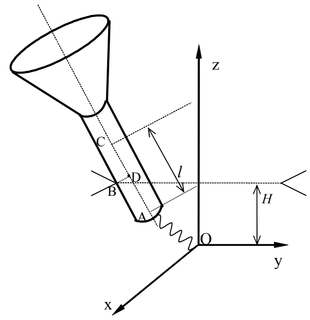

Fig. 3. Calculation scheme.

Let’s denote point of contact between the rotor and the hub B (see Fig. 3) and drop a perpendicular from point B to axis of the rotor. Crossing of these lines we will call point D. It seems reasonably to describe motion of the rotor with respect to fixed coordinate system ($Oxyz$) using coordinates of point D (x_D, y_D), vertical displacement of the rotor from the state of static equilibrium (z) and Euler’s angles ψ, θ, φ (ψ is a precession angle, θ and φ are angles of nutation and proper rotation). We consider nutation angle θ to be small, i.e. $\cos\theta \approx 1$, $\sin\theta \approx 0$.
The peculiarity of the system is the presence of permanent contact between the rotor and the hub. When the rotor moves always staying in contact with the hub the following relationship is valid:

$$x_D^2 + y_D^2 = \varepsilon^2, \tag{1}$$

where $\varepsilon = R - r$, R and r are radiuses of the hub and the rotor shaft, ε is a small variable. Using (1) we can introduce a new variable α instead of x_D and y_D:

$$x_D = \varepsilon\cos\alpha, \qquad y_D = \varepsilon\sin\alpha . \tag{2}$$

Equations of motion for the rotor could be written in the following form:

$$M\varepsilon\ddot\alpha - (Mh\ddot\theta - Mh\theta\dot\psi^2 - k_1H\theta)\cos(\psi-\alpha) + Mh(\theta\ddot\psi + 2\dot\theta\dot\psi)\sin(\psi-\alpha) = F_{\text{fr}\,y}\cos\alpha - F_{\text{fr}\,x}\sin\alpha,$$
$$M\ddot z + k_2 z = F_{\text{fr}\,z},$$
$$A\ddot\theta + C\theta\dot\psi(\dot\varphi+\dot\psi) - A\theta\dot\psi^2 + h(Mh\ddot\theta - Mh\theta\dot\psi^2 - k_1H\theta)\sin^2(\psi-\alpha) - \varepsilon(Mh\dot\alpha^2 + k_1H)\sin(\psi-\alpha) - (Mg - k_1H)l\theta +$$
$$+ Mh^2(\theta\ddot\psi + 2\dot\theta\dot\psi)\sin(\psi-\alpha)\cos(\psi-\alpha) = h[F_{\text{fr}\,x}\cos\alpha + F_{\text{fr}\,y}\sin\alpha]\sin(\psi-\alpha) + m_\xi^{(C)}(F_{\text{fr}})\cos\varphi - m_\eta^{(C)}(F_{\text{fr}})\sin\varphi, \qquad (3)$$
$$A\ddot\psi\theta - C\dot\theta(\dot\varphi+\dot\psi) + 2A\dot\theta\dot\psi - \varepsilon(Mh\dot\alpha^2 + k_1H)\cos(\psi-\alpha) + h(Mh\ddot\theta - Mh\theta\dot\psi^2 - k_1H\theta)\sin(\psi-\alpha)\cos(\psi-\alpha) +$$
$$+ Mh^2(\theta\ddot\psi + 2\dot\theta\dot\psi)\cos^2(\psi-\alpha) = h[F_{\text{fr}\,x}\cos\alpha + F_{\text{fr}\,y}\sin\alpha]\cos(\psi-\alpha) + m_\xi^{(C)}(F_{\text{fr}})\sin\varphi + m_\eta^{(C)}(F_{\text{fr}})\cos\varphi,$$
$$C(\ddot\varphi+\ddot\psi) = m_\zeta^{(C)}(F_{\text{fr}})\,.$$

Here M is mass of the rotor; $h = l - H$ (see Fig. 3); k_1 and k_2 are rigidities of supporting elastic elements; A and C are moments of inertia of the rotor; $\boldsymbol{F}_{\text{fr}}$ is a force of friction acting on the rotor; $\boldsymbol{m}^{(C)}(F_{\text{fr}})$ is a moment of this force with respect to the center of mass C. A force of normal reaction N acting in point B is eliminated from these equations, it is determined by the following relationship:

$$N = \varepsilon(M\dot\alpha^2 - k_1) - (Mh\ddot\theta - Mh\theta\dot\psi^2 - k_1H\theta)\sin(\psi-\alpha) - Mh(\theta\ddot\psi + 2\dot\theta\dot\psi)\cos(\psi-\alpha) + F_{\text{fr}\,x}\cos\alpha + F_{\text{fr}\,y}\sin\alpha.. \qquad (4)$$

The system of equations (3) has 10$^{\text{th}}$ order and is essentially nonlinear even on the assumption that θ is small angle.
In the case when force of friction F_{fr} depends on normal reaction the equations (3) and (4) could be rewritten in a slightly different way.

STATIONARY MOTION OF THE ROTOR AND ITS STABILITY

Equations (3) have the following stationary solution:

$$\begin{array}{lll} \alpha = \omega t, & z = 0, & \\ \psi = \omega t + \dfrac{\pi}{2}, & \theta = \theta_0 = const\,, & \varphi = -\Omega t, \end{array} \qquad (5)$$

which corresponds to the motion of the rotor without sliding with constant angular velocities of precession (ω) and proper rotation (Ω) and constant nutation angle.
It is shown that such motion does not depend on friction law in the system. It exists in the case of absolutely smooth surface ($\boldsymbol{F}_{\text{fr}} = 0$) as well as in the case of dry and viscous friction in point B. Parameters of the stationary motion θ_0 and N_0 are determined from the equations (3) and (4) when (5) is taken into account. In Fig. 4 transition motion for the case of viscous friction is presented. It can be seen that velocity of point B (curve a) tends to zero, i.e. motion without sliding begins; angle of nutation (curve b) approaches some constant, and difference between angle velocities $\alpha'(t)$ and $\psi'(t)$ (curve c) also tends to zero, so the system approaches the motion (5).

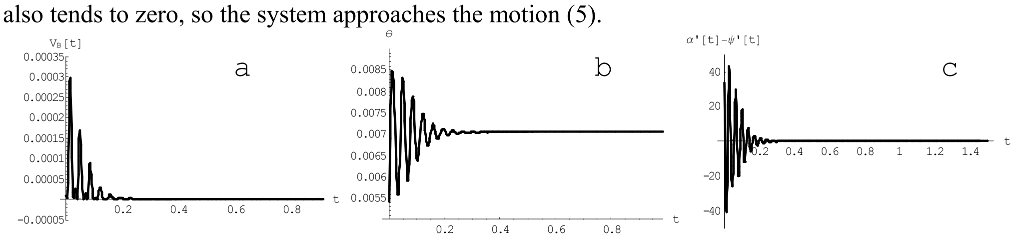

Fig. 4. Transient motion for the case of viscous friction.

Analysis of stability of the motion (5) in a conservative system, when force of friction is not taken into account, allows to obtain conditions which are necessary and sufficient for stability analytically. Together with these rather cumbersome relationships the condition which can be written in a more simple form is obtained:

$$k_1 H^2 > (A + Mh^2)\omega^2 + Mgl\,. \qquad (6)$$

It is proved that this condition is sufficient for stability of the stationary solution. It can be seen that stability of the motion in considered system is not of a gyroscopic type and thus forces of friction could not break stability [1].

CONCLUSIONS

Investigation of stability of stationary motion in considered system rotor–stator shows that together with rather cumbersome conditions of stability the simple condition sufficient for stability could be obtained. Considered motion exists in the system in conservative case as well as in the case when forces of dry and viscous friction are taken into account. It is shown that stability does not have gyroscopic character and thus forces of friction could not break stability.
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References

[1] Merkin D. R.: Introduction to the theory of stability of motions. M.: Nauka, 1971. 312 p. (In Russian)