# THE NEW STATEMENT OF PROBLEM OF UNBALANCE IDENTIFICATION

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*Summary* The algorithms of inverse problem solution of rotor unbalance identification in new statement is offered. This statement of problem permits to obtain the most probable solution. The vibrations of rotor supports in two mutually perpendicular directions during the work for a few rotor rotations as the initial information are used. Tikhonov regularization method is applied for solution of this ill-posed (unstable) problem taking into consideration the error of mathematical model. The numerical calculations of examples are given to illustrate these algorithms.

## PROBLEM DEFINITION

Let us consider a deformable rotor rotating on two no region supports [1]. The motion of rotors described by system of ordinary differential equations of 18th order [2,3]. The rotor unbalance is modelling by some external load. The value of this load and the place of its action is it necessary to find. It is assumed that the vibrations of rotor supports in two mutual perpendicular directions are obtained from experiment. Let us suppose that the functions $z_1(t)$, $z_2(t)$, $z_3(t)$ characterize the unbalance of rotor $z_1(t) = m_r r\, \dot{\varphi}^2 \, Sin(\vartheta+\varphi)$, $z_2(t) = m_r r\, \dot{\varphi}^2 \, Cos(\vartheta+\varphi)$, $z_3(t) = hr\, \dot{\varphi}^2 \, Sin(\vartheta+\varphi)$, (r is the radius of rotor, $m_r$ is the mass of unbalance reducing to a surface of rotor, $\dot{\varphi}$ is the angular velocity of rotation, h is unbalance arm, $\vartheta$ is angular deviation of the factor of unbalance with respect to correction plane). We suppose that with the help of acceleration transducers the function $\ddot{\xi}_A(t)$, $\ddot{\xi}_B(t)$, $\ddot{\eta}_A(t)$, $\ddot{\eta}_B(t)$ have been recorded ( $\ddot{\xi}_A(t)$, $\ddot{\xi}_B(t)$ are the acceleration of supports A and B in horizontal direction, $\ddot{\eta}_A(t)$, $\ddot{\eta}_B(t)$ are the acceleration of supports A and B in vertical direction). Then the problem of unbalance recognition is reduced to the solution of Volltera integral equations of first kind (as an example, we consider the equation for the unknown function $z_1(t)$ only).

$$\int_0^t (t-\tau)\, z_1(t)\, dt = u_1(t), \;\; t\in [0,T] \;\text{ or }\; A z = B_{\mathbf{p}}\, \mathbf{x}_\delta = u_\delta, \tag{1}$$

where A is a linear operator($A: Z \to U$; Z,U are the functional B- spaces), z is the searched characteristic of unbalance, $B_{\mathbf{p}}$ is a linear irreversible operator ($B_{\mathbf{p}}: X \to U$; X is the functional B- space) depending on parameters vector of MM of "rotor-supports" $\mathbf{p} = (p_1, p_2, \ldots, p_n)^*$ ($(.)^*$ is the mark of transposition); $\mathbf{x}_\delta = (\ddot{\xi}_A(t), \ddot{\xi}_B(t), \ddot{\eta}_A(t), \ddot{\eta}_B(t))^*$ is the vector-function of initial data.

Subjective factors influence on the definition of parameters of system "rotor-supports" MM and therefore the parameters are supposed to have their values within certain limits: $p^0_i \le p_i \le \hat{p}_i$, $1 \le i \le N$. In this way the vector $\mathbf{p}$ can be changed inside the known closed region $\mathbf{p} \in \mathbf{D} \subset R^N$.

The equations for required functions $z_2(t)$, $z_3(t)$ will be similar to the equation (1).

The vector-function $\mathbf{x}_\delta$ is obtained using the experimental data (vibrations of supports) where the noise is present. Therefore it is convenient to think that each component of vector-function $\mathbf{x}_\delta$ and function $u_\delta$ belong to $L_2[0,T]$. Under this conditions the problem of equation (1) solution is ill-posed problems if the searched functions $z_i(t)$ belong to C[0,T] [4]. The value of function deviation $u_\delta$ from the exact function $u_T$ is:

$$\| u_T - u_\delta \|_U = \| B_p\, \mathbf{x}_\delta - B_T\, \mathbf{x}_T \|_U \le \delta_0,$$

where $\delta_0 = \delta\, b_0 + d \, \| \mathbf{x}_\delta \|_X$, $b_0 = \sup_{\mathbf{p}\in\mathbf{D}} \| B_{\mathbf{p}} \|$, $\| B_T - B_{\mathbf{p}} \|_{X\to U} \le d$, $\| \mathbf{x}_\delta - \mathbf{x}_T \|_X \le \delta$, $\mathbf{x}_T$ is the exact vector- function of initial data; $B_T$ is the exact operator; $\delta$, $b_0$, d are given values. Let us suppose that such exact solution of equation (1) $\mathbf{z}_T$ ($A\, \mathbf{z}_T = u_T = B_T\, \mathbf{x}_T$ ) belongs to the function space $W^1_2[0,T]$ [4].

The regularization method of Tikhonov is used for definition of stable approximate solution [4]. This way is based on the search of following problem solution [2,3,5]:

$$\Omega[\tilde{z}] = \inf_{z\in Q_{d;\delta}} \Omega[z], \tag{2}$$

where $Q_{d,\delta}$ is the set of possible solutions of equation (1) with the error of calculation

$$Q_{d,\delta} = \{\, z : z \in Z_1 \subset Z ,\; \| A z - B_{\mathbf{p}}\, \mathbf{x}_\delta \|_U \le \delta_o \},$$

$\Omega[z]$ is the stabilizing functional which is defined on $Z_1$ ($Z_1$ is the everywhere dense set into Z).

From the practical point of view the function $\tilde{z}$ gives a guaranteed estimation from below sizes of real of a rotor in sense of functional $\Omega[z]$. If $\Omega^0 \le \Omega[\tilde{z}]$ ($\Omega^0$ there is known limiting an allowable value for the given type of rotor machine),

then the rotor works in emergency operation with guarantee. If the inequality $\Omega[\tilde{z}] < \Omega^0$ is carried out, then no objective conclusions can be made.
Let us consider the sets:

$$Q_{\mathbf{p},\delta} = \{ z : z \in Z_1,\ \| A z - B_{\mathbf{p}} x_\delta \|_U \le \delta \| B_{\mathbf{p}} \| ,\quad Q^* = \bigcup_{\mathbf{p}\in\mathbf{D}} Q_{\mathbf{p},\delta} \ (\cup \text{ is the union}).$$

In the given work the new statement of a problem of rotor unbalance identification is suggested: to find the function $z_{\mathbf{p}}^r$ among set of the possible solutions of the equation (1) which would give the least maximal deviation from the experimentally measured vibrations of support of a rotor for all operators $B_{\mathbf{p}}$. Such statement is reduced to the solution of the following extreme problem:

$$\| A z_{\mathbf{p}}^r - B_{\mathbf{p}}^r \mathbf{x}_\delta \|_U = \inf_{z_{\mathbf{p}}} \sup_{B_{\mathbf{p}}} \| A z_{\mathbf{p}} - B_{\mathbf{p}} \mathbf{x}_\delta \|_U, \tag{3}$$

where $\Omega[z_{\mathbf{p}}] = \inf_{z\in Q_{\mathbf{p};\delta}} \Omega[z]$.

As all operators $B_{\mathbf{p}}$ it is possible to consider equivalent within the limits of the specified accuracy, it is possible to consider function $z_{\mathbf{p}}^r$ as the most probable solution of a problem of unbalance identification. The function $z_{\mathbf{p}}^r$ will be call the *most probable function of unbalance*. The most probable solution $z_{\mathbf{p}}^r$ will coincide with classical regularizing the solution of an inverse problem of unbalance identification of a rotor if there is the one operator $B_{\mathbf{p}}$ only. The function $z_{\mathbf{p}}^r$ is the best approximation of real unbalance and also is steady to small deviations of the initial data.
For suggested algorithm examination of unbalance characteristics evaluation there was calculated the case when functions $\ddot{\xi}_A(t)$, $\ddot{\xi}_B(t)$, $\ddot{\eta}_A(t)$, $\ddot{\eta}_B(t)$ are the results of mathematical simulation of rotor vibrations with given unbalance. The value of rotor unbalance were chosen as:

$$m_r = 0.5 \text{ kg by } r = 0.25 \text{ м}, h = 0.25 \text{ м}, \vartheta = 0.5 \text{ rad}.$$

The values of initial data inaccuracy were chosen as the following:

$$\| \ddot{\xi}_A(t) - \ddot{\xi}^T_A(t) \|_C \le \delta_1 = 0.08 \frac{\text{м}}{\sec^2},\ \| \ddot{\xi}_B(t) - \ddot{\xi}^T_B(t) \|_C \le \delta_2 = 0.1 \frac{\text{м}}{\sec^2},\ \| \ddot{\eta}_A(t) - \ddot{\eta}^T_A(t) \|_C \le \delta_3 = 0.1 \frac{\text{м}}{\sec^2},$$

$$\| \ddot{\eta}_B(t) - \ddot{\eta}^T_B(t) \|_C \le \delta_4 = 0.1 \frac{\text{м}}{\sec^2}.$$

The results of identification of most probable function of unbalance are the following: $m_r = 0.42$ kg by r = 0.23 м, h = 0.22 м, $\vartheta = 0.48$ rad. The discrepancy method defined the parameter of regularization $\alpha$ [4].

## CONCLUSIONS

The suggested statement of unbalance identification permits to evaluate all characteristics of rotor unbalance in real time with maximum exactness. It can be used for technical diagnostics of unbalance and for balancing of rotors in their own bearings. The method can be adapted to those cases when measuring velocity or displacement of supports.

## References

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