SOME INVESTIGATIONS ON FM BEM IN SOLID MECHANICS

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Summary A new version algorithm of $O(N)$ FM BEM for 2D elastostatics is presented and applied to the simulation of 2D elastic solid with large number of randomly distributed inclusions. Furthermore, FM BEM is applied to the simulation of 2D elastic solid containing large number of randomly distributed cracks.

INTRODUCTION

As well known, the boundary element methods (BEM) has remarkable advantages of dimension reduction and higher accuracy, but the conventional BEM is not efficient for large scale problems. For the conventional BEM $O(N^3)$ operations and $O(N^2)$ memory are required, where N is the number of DOF.

Fast multipole (FM) methods presented by Rokhlin, Greengard and other mathematicians have reduced the operations and memory requirement tremendously, finally reduced to $O(N)$. The FM BEM becomes an attractive way to solve complex practical scientific and engineering problems with BEM. There are nearly 500 papers so far have been cited by SCI in the field of fast multipole BE. In solid mechanics, especially if the local effects are important in the analysis, the higher accuracy is required, and some further investigations are still required for their application in different large scale problems.

In this presentation, some investigations of authors' group on FM BEM in solid mechanics are presented, including: a new version algorithm of $O(N)$ FM BEM for 2D elastostatics and its application to the simulation of 2D elastic solid with large number of randomly distributed inclusions; application of FM BEM to the simulation of 2D elastic solid containing large number of randomly distributed cracks. Three years ago in authors' group, the largest scale of practical problem solved by conventional BEM was 54,000 DOF, on a parallel PC cluster consisted of 8 PCs. The scale of numerical examples by FM BEM presented in this presentation, already reaches 544,000 DOF and 1,500,000 DOF, calculated on one PC.

A NEW VERSION OF FM BEM FOR 2D ELASTOSTATICS

Original FM BEM of $O(N)$

The **Step 1** of FM BEM for 2D elastostatics is *Multipole Expansion of the Kernels*. The far-field expansion of kernels is performed in terms of complex Taylor series with respect to the field point y around a selected point y_0, which is the center of a small square box A containing y, as shown in Fig.1. The **Step 2** is *Shift of Multipole to Multipole Expansion*. The multipole expansion with respect to the center of smaller square box A is shifted to that with respect to point y_1, the center of larger square box B. The **Step 3** is *Shift of Multipole to Local Expansion*. The multipole expansion with respect to the center of square box B is shifted to the local expansion with respect to point x_0, the center of square box C, which is as the same size as the box B, but containing the source point x. The **Step 4** is Shift of Local to Local Expansion. The local expansion with respect to the center of square box C is shifted to that with respect to point x_1, the center of a smaller square box D including x. The details of the algorithm and the formulas can be found in Ref. [1].

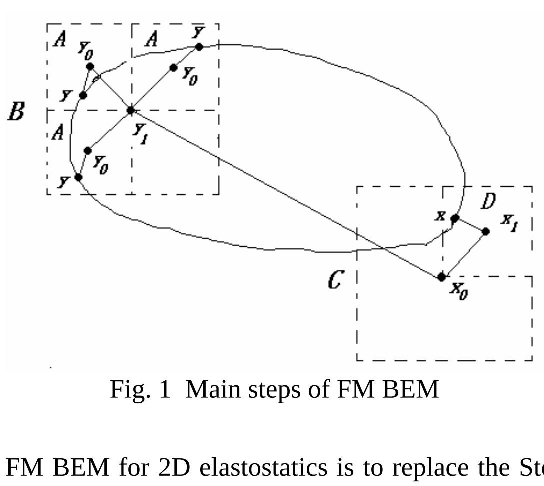
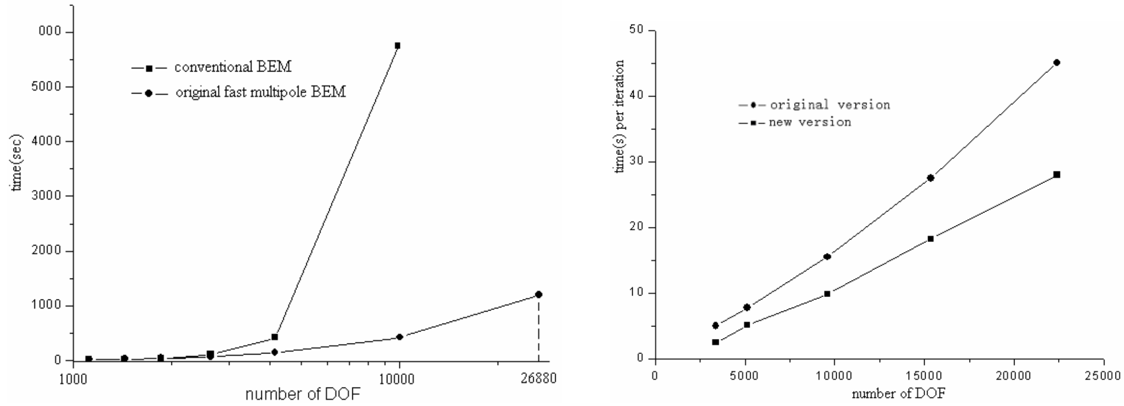


Fig. 1 Main steps of FM BEM

New Version FM BEM

The main idea of the new version FM BEM for 2D elastostatics is to replace the Step 3 of original one by other three sub-steps, which make the translation operator diagonal. The **Step 3.1** is *Shift of Multipole to Exponential Expansion*; the **Step 3.2** is *Shift of Exponential to Exponential Expansion*; and the **Step 3.3** is *Shift of Exponential to Local Expansion*. Although it is more complex, the efficiency is further improved. The comparison of conventional BEM, original FM BEM and new version FM BEM in computational speed can be found in Fig. 2.



a) b)

Fig. 2 Comparison of conventional BEM, original and new version FM BEM in computational speed

SIMULATION OF 2D ELASTIC SOLID WITH LARGE NUMBER OF INCLUSIONS

The FM BEM has been successfully applied to the computation of the effective properties of 2D elastic solid with 1600 randomly distributed circular inclusions as shown in Fig.3. Actually, for such problem, the sample with 100 inclusions is sufficient, and it can be computed by conventional BEM. But the example shown here with 544,000 DOF is a large scale problem, which can not be treated using conventional BEM. For the further investigation of 3D composites it will be a large scale problem as well, only the FM BEM has potential to simulate it successfully.

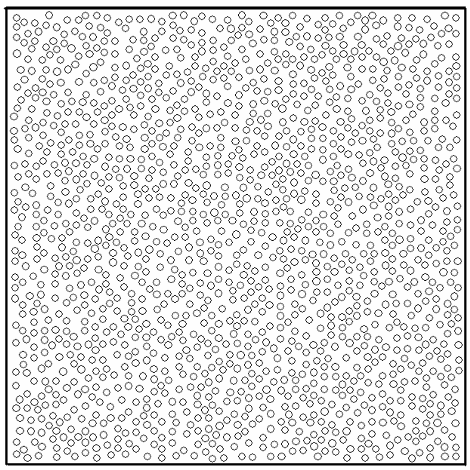


Fig. 3 2D elastic solid with 1,600 randomly distributed inclusions

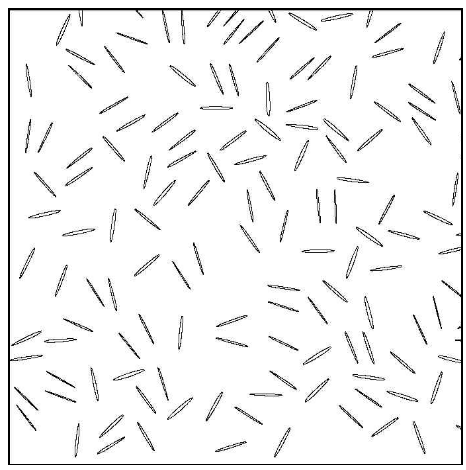


Fig. 4 zoomed deformation pattern of 2D solid containing large number of cracks

SIMULATION OF 2D ELASTIC SOLID WITH LARGE NUMBER OF CRACKS

A new form of complex Taylor series is applied to reduce the number of multipole moments. An improved preconditioner for GMRES is developed for reducing computing time and memory requirement. The FM BEM has also been successfully applied to the computation of 2D infinite elastic solid containing 16384 randomly distributed cracks in a finite region, with 1,572,864 DOF. The computing time on one PC is about 7 hours. The zoomed deformation pattern is shown in Fig. 4.

CONCLUDING REMARKS

The investigations on FM BEM in simulation of 2D elastic solids have shown its attractive advantages. The FM BEM can be applied to some large scale problems. The further investigation in authors' group will be concentrated on 3D elasticity problems, including composites and thin structures, and on the simulation of the failure process of the composite materials.

References

[1] Wang, H.T., Yao, Z.H.: Simulation of 2D elastic solid with large number of inclusions using fast multipole BEM. In: Bathe, K.J. ed. *Computational Fluid and Solid Mechanics 2003*, 732-736, Elsevier, 2003.