OPTIMIZATION OF BEAM PROPERTIES WITH RESPECT TO MAXIMUM BAND-GAP

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Summary We study numerically the frequency band-gap phenomenon for bending waves in an infinite periodic beam. The outcome of the analysis is then subjected to an optimization problem in order to maximize these band-gaps. The band-gap maximization may be performed with respect to material parameters and cross-sectional geometry.

INTRODUCTION

The study of frequency band-gaps in certain material structures subjected to wave propagation has received a lot of attention during the last years. Such materials hinder wave propagation with frequencies in certain intervals (band-gaps). The phenomenon occurs for both acoustic, elastic and electromagnetic waves. Examples among numerous work include [1] which analyzes band-gaps for electromagnetic waves in infinite 2D structures, [2] carries out both an analysis and optimization for the elastic 2D structural problem (infinite and finite), [3] considers infinite periodic plates and cylindrical shells with and without loading. Such studies are relevant in structural design regarding e. g. vibration damping and filters.

In this article, we first perform a numerical analysis of frequency band-gaps for time-harmonic bending waves in an infinite periodic beam. We then formulate an optimization problem for maximizing these band-gaps.

THEORY

Analysis

Bending waves in an infinite periodic beam has been analyzed theoretically using Floquet theory in [4]. Here we adopt the finite element method based on Floquet theory described in [1] to derive the eigenvalue problem for studying the frequency band-gaps.
Bending waves in a beam are described by the fourth order beam equation. We restrict our attention to waves with a time-harmonic behavior $\mathrm{e}^{\mathrm{i}\omega t}$. The differential equation describing the transverse displacement $w(x)$ then becomes

$$\frac{d^2}{dx^2}(E(x)I(x)\frac{d^2w(x)}{dx^2}) = \omega^2\rho(x)A(x)w(x) \tag{1}$$

where E, I, ρ and A are Young's modulus, moment of inertia, mass density and cross-sectional area respectively. Here we consider a periodically segmented beam of infinite length as shown in Fig. 1. The beam consists of an infinite number of identical base cells of length d. Each cell is made up of two different materials with possibly varying cross-section as indicated. Thus, $E(x) = E(x+d)$ and analogously for ρ, I, and A. In order to study the problem, we assume that the

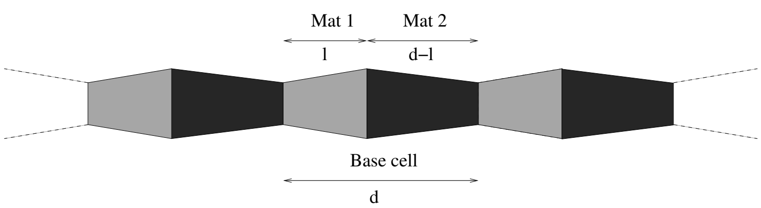

Figure 1. Periodic infinite beam with identical base cells consisting of two materials (Mat 1 and Mat 2) with possibly varying cross-sections.

transverse displacement can be written in terms of Bloch functions

$$w(x) = \mathrm{e}^{ikx}u(x) \tag{2}$$

where k is the wave number and $u(x) = u(x+d)$. Inserting this assumption and integrating over the length of the base cell we obtain

$$-\int_0^d E(x)I(x)(\frac{d}{dx}+\mathrm{i}k)^2u(x)\overline{(\frac{d}{dx}+\mathrm{i}k)^2v(x)}\,\mathrm{d}x = \omega^2\int_0^d \rho(x)A(x)u(x)\overline{v(x)}\,\mathrm{d}x$$

where $v(x)$ is an arbitrary periodic function on the base cell. We use a standard shape function assumption based on a third order polynomial $\phi_i(x)$ on each element, such that $u(x) = \sum_i u_i\phi_i(x)$. We then arrive at the following complex eigenvalue problem

$$\mathbf{K}(k)\mathbf{u} = \lambda\mathbf{M}\mathbf{u}, \qquad \lambda = \omega^2. \tag{3}$$

where

$$K_{ij} = -\int_0^d E(x)I(x)(\frac{d}{dx}+\mathrm{i}k)^2\phi_i(x)\overline{(\frac{d}{dx}+\mathrm{i}k)^2\phi_j(x)}\,\mathrm{d}x, \qquad M_{ij} = \int_0^d \rho(x)A(x)\phi_i(x)\overline{\phi_j(x)}\,\mathrm{d}x$$

The solution to this problem results in the dispersion relation

$$\omega_j = \omega_j(k), \qquad j = 1,\dots,\text{d.o.f} \tag{4}$$

where ω_i denotes the i'th frequency band and d.o.f. denotes the number of independent variables.

Optimization

The optimization problem for the band-gaps can now be written as a max-min objective using the dispersion relation [2]

$$\max_{\rho,E,A} : c(\rho,E,A) = \frac{\Delta\omega^2(\rho,E,A)}{\omega_0^2(\rho,E,A)} = 2\frac{\min_k \omega_{j+1}^2(k,\rho,E,A) - \max_k \omega_j^2(k,\rho,E,A)}{\min_k \omega_{j+1}^2(k,\rho,E,A) + \max_k \omega_j^2(k,\rho,E,A)} \tag{5}$$

RESULTS

Here we present results for an infinite beam with constant cross-section area $A = 7.74\times10^{-4}\,\mathrm{m}^2$, $I = 4.16\times10^{-8}\,\mathrm{m}^4$, $d = 0.254\,\mathrm{m}$ and $l = d/2 = 0.127\,\mathrm{m}$. Dispersion plots including the first five frequency bands are shown in Fig. 2 for two material combinations shown in Table 1 (row 1 and 2 respectively). In both cases Mat 1 is Zinc, while Mat 2 is chosen to

Mat 1	E_1 (GPa)	ρ_1 (kg/m^3)	Mat 2	E_2 (GPa)	ρ_2 (kg/m^3)
Zinc	98	7130	Silicon	100	2330
Zinc	98	7130	Chromium	25	7200

Table 1. Material parameters (rows) for two different base cells .

obtain differences in either E or ρ. It is seen that both material combinations result in relative band-gaps approximately the same size but at different frequencies. This is to be studied more thoroughly using (5).

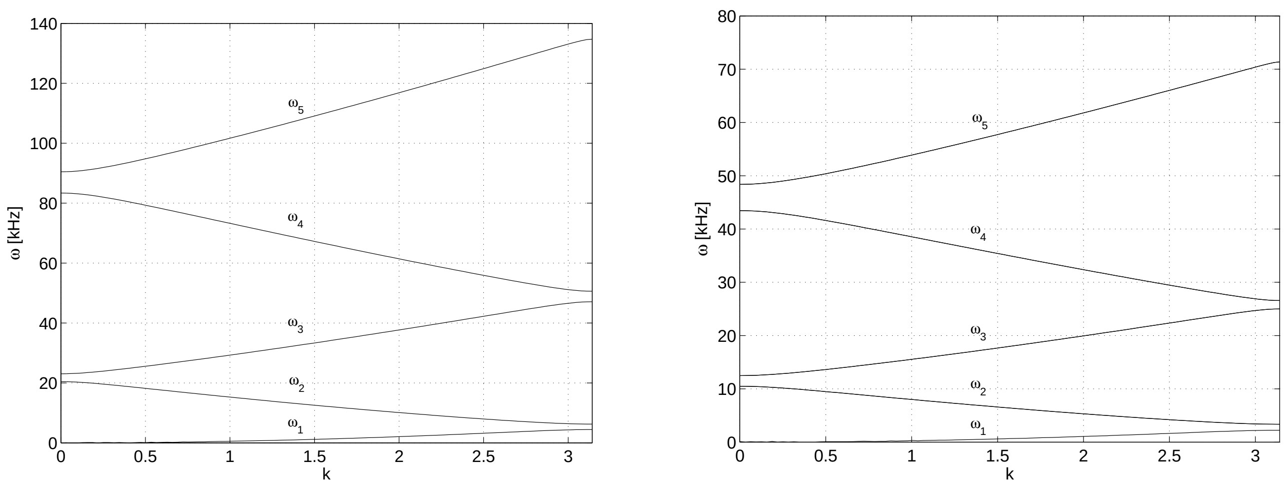

Figure 2. Dispersion plots including the first five frequency bands showing the band-gaps for two base cells with different material combinations. Left: Base cell consisting of Zinc-Silicon. Right: Base cell consisting of Zinc-Chromium.

ACKNOWLEDGMENTS

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