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MATHEMATICAL MODELS OF MICROCONVECTION FOR ISOTHERMALLY INCOMPRESSIBLE AND WEAKLY COMPRESIBLE LIQUIDS

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Summary The term "microconvection" was originally introduced to characterize non-solenoidal flows driven by density (depending on temperature only) changes. These phenomena were analyzed for thermal buoyancy-driven convection (Pukhnachov, 1991) and for diffusive-induced flows (Perera and Sekerka, 1997). The effect on non-solenoidality is equally important for the description of non-stationary convection in microgravity conditions and in microscales. Recently this approach was extended up to convective motions of a weakly compressible liquid (Pukhnachov, 2002). On the basis of the microconvection model, there are studied a number of problems describing the convection in a vertical layer with the thermal fluxes oscillating in a phase and in an anti-phase, flows in a circular ring and in a prolate rectangular, mixed thermocapillary/gravitational convection in a semicircle with a free flat boundary.

We consider the mathematical models for the thermal gravitational convection assuming smallness of the microconvection parameter, which is equal to ratio of the velocity orders produced by liquid expansion and buoyancy factor. They are the model of microconvection for isothermally incompressible liquid known since 1991 (see full presentation in [1], Chapter VI) and the model for weakly compressible liquid [2]. Characteristic property of both models is the non-solenoidal velocity field. The parameter of microconvection is an additional similarity criterion regarding to Grashoff and Prandtl numbers. It is equal to gravity acceleration multiplied by cube of the characteristic linear scale and divided by the product of the kinematic viscosity and the thermodiffusion coefficient. Thus, we study the convective fluid flows under low gravity, in small scales and at fast changes of the boundary thermal regimes.

At first, the model of microconvection was proposed and used for the analytical and numerical research of the convection in a closed cavity under low gravity. This model is based on the assumption that the liquid density depends on its temperature only. It is demanding that the total heat flux through the boundary of a closed cavity should be equal to zero at each moment of time. The model of weakly compressible liquid is free of this deficiency. The liquid is considered now as a thermodynamic medium with two state parameters: temperature and pressure.

The model of weakly compressible liquid includes systems of exterior and interior expansions in time. Since the equations of the exterior expansion are not evolutional with respect to pressure, we should construct an interior expansion to compensate the discrepancy in the initial condition for pressure. This problem is solved in a linear approximation.

We develop the ideas from [1, 2], explain the definitions of the isothermally incompressible and weakly compressible liquids, give an analysis of the criteria of similarity and of the characteristic values for the convective flows and present the analytical and numerical results for some mathematical problems.

Taking into account the non-solenoidality for the stationary problems of microconvection in the closed domains leads to the corrections of the orders of Boussinesq number (V.V. Pukhnachov, 1996; O.N. Goncharova, 1997).

The qualitative differences in flow characteristics for the stationary problems with free boundary in a semicircular region are observed, when boundary thermal regimes have local singularity (O.N. Goncharova, 2001).

The most bright qualitative and quantitative differences from the classical results or so-called non-Boussinesq flow effects are obtained by the simulations for the non-stationary problems of microconvection: convection in an annular domain at time dependent microgravity forces (O.N. Goncharova, 1995, 1997); convection in a vertical layer (V.V. Pukhnachov, 1994; O.N. Goncharova, 2000); convection in a long rectangle at the periodic boundary thermal regimes(O.N. Goncharova, 2000); convection in a vertical layer with the thermal fluxes, which oscillate in a phase and in an anti-phase (O.N. Goncharova, 2004).

Our mathematical modeling of the convection can be called alternative approach in the convection theory. In fact, we are starting from the exact conservation laws of mass, momentum and energy and are building an hierarchy of the models in the convection theory: the Navier-Stokes equations for compressible fluid ă the model for weakly compressible fluid ă the model of microconvection ă the Oberbeck-Boussinesq model.

From the mathematical point of view, the alternative theories of the convection of fluids were developed by J.M. Mihaljan (1962), V.I. Yudovich (1983, 1999), K.A. Nadolin (1989, 1995). The stability of the solutions in the microconvection model was studied by V.K. Andreev (since 1996). P.S. Perera and R.F. Sekerka (1997) proposed the non-solenoidal model for the concentration convection analogous to the microconvection model. The investigation of mathematical problems in the non-solenoidal model of convection was done by V.B. Moseenkov (1998).

References

[1] Andreev V.K., Kaptsov O.V., Pukhnachov V.V., Rodionov A.A.: Applications of Group-Theoretical Methods in Hydrodinamics. Kluwer Academic Publ., Dordrecht/Boston/London, 1998.

[2] Pukhnachov V.V., Hierarchy of the models in the convection theory. *Notices of Petersburg Division of Steklov Mathematical Institute* **288**: 152–177, 2002.