OPTIMAL LAYOUT OF TWO MATERIALS WITHIN THE CORE LAYER OF A SANDWICH PLATE. RELAXED FORMULATION AND ITS COMPUTATIONAL ALGORITHM

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Summary The paper deals with the minimum compliance problem of a sandwich plate with soft core. The aim is to find an optimal layout of two isotropic materials within the core layer. In the relaxed formulation the core is modelled by a 1st rank laminated composite.

Deformation of a sandwich plate of a soft core, see Fig.1, transversely loaded, will be modelled by the Reissner model of 1947, see Sec. 6.3 of Ref.[5]. The bending stiffnesses $\left(D^{\alpha\beta\lambda\mu}\right)$, $\alpha,\beta,\lambda,\mu=1,2$, depend on the properties of the faces and on the distance $2c$. The aim is to optimize the core the transverse shear moduli of which determine the effective stiffnesses $\left(H^{\alpha\beta}\right)$. The moments $\left(M^{\alpha\beta}\right)$ and the transverse forces $\left(Q^{\alpha}\right)$ are interrelated with the deformation measures by

$$M^{\alpha\beta}=D^{\alpha\beta\lambda\mu}\varepsilon_{\lambda\mu}(\boldsymbol{\varphi}),\quad Q^{\alpha}=H^{\alpha\beta}\gamma_{\beta}(w,\boldsymbol{\varphi}) \tag{1}$$

where $\boldsymbol{\varphi}=(\varphi_1,\varphi_2)$ is a vector of the angles of rotation of the transverse cross sections and w represents the plate deflection; the deformations $\varepsilon_{\lambda\mu}(\boldsymbol{\varphi})$ are defined as symmetric part of the gradient of $\boldsymbol{\varphi}$ and $\gamma_{\beta}(w,\boldsymbol{\varphi})=w,_{\beta}+\varphi_{\beta}$. Here comma means differentiation with respect to x_1 or x_2.

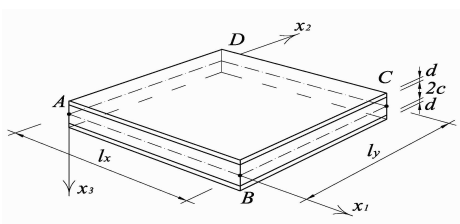

Fig.1 Sandwich plate

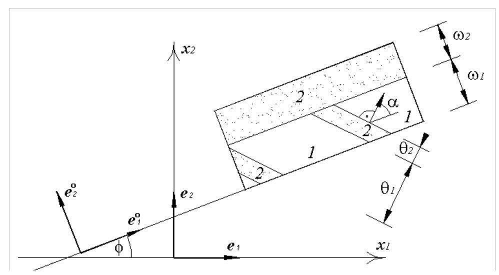

Fig.2 General 2nd rank microstructure of the core layer

The variational equilibrium equation of the plate has the form

$$\int_{\Omega}\left[\, M^{\alpha\beta}\varepsilon_{\alpha\beta}(\overline{\boldsymbol{\varphi}})+Q^{\alpha}\gamma_{\alpha}(\overline{w},\overline{\boldsymbol{\varphi}})\,\right]dx=f(\overline{w})\qquad\forall(\overline{w},\overline{\boldsymbol{\varphi}})\in V\qquad f(\overline{w})=\int_{\Omega}q(x)\overline{w}(x)dx \tag{2}$$

where q represents the transverse loading, Ω is a plate middle plane and V represents the space of kinematically admissible displacement fields. Assume that the core is formed transverse homogeneously of two kinds of isotropic core materials labelled by $\gamma=1,2$. Assume that these materials occupy the domains Ω_{γ} within Ω; the characteristic functions of the domains being denoted by χ_{γ}. Let the areas $\left|\Omega_{\gamma}\right|=C_{\gamma}$ be prescribed. The problem of minimizing the compliance of the plate is put in the form

$$\min\left\{f(w)\middle|\, w\quad\text{solves (1), (2) and } \left|\Omega_2\right|=C_2\right\} \tag{3}$$

The relaxation of the above problem means admitting the characteristic functions χ_{γ} to tend to their weak limits $m_{\gamma}\in L^{\infty}(\Omega;[0,1])$, see Díaz et al.[4] and Lewiński and Telega [5]. Let $G_{m_2}^{per}$ be a set of tensors $\mathbf{H}^{*}$ determining the transverse shear stiffness of the composite material of the core, of two-phase periodic structure, according to the formulae of the homogenization theory, under the assumption of the area fraction of the material 2 being given as m_2. Let us define the effective potential for the transverse forces

$$2W_s(\boldsymbol{\gamma},m_2)=\max\left\{\boldsymbol{\gamma}:(\mathbf{H}\boldsymbol{\gamma})\,\middle|\,\mathbf{H}\in G_{m_2}^{per}\right\} \tag{4}$$

and note that its complete characterization is known by analogy with the conductivity problem. Thus the effective constitutive relationships read

$$\mathbf{M}=\mathbf{D}\boldsymbol{\varepsilon}(\boldsymbol{\varphi})\ ,\qquad \mathbf{Q}=\frac{\partial W_s(\boldsymbol{\gamma},m_2)}{\partial\boldsymbol{\gamma}} \tag{5}$$

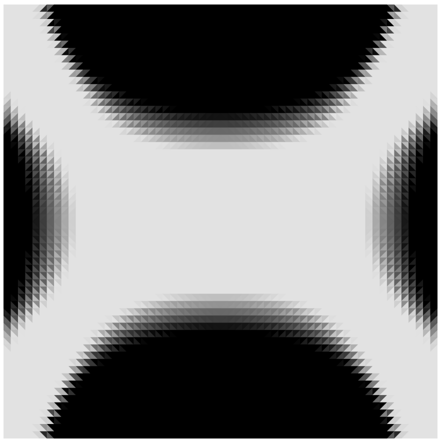

Fig.3 Optimal distribution of the area fraction m_2

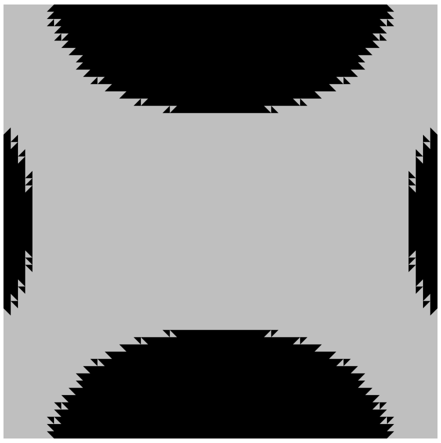

Fig. 4 Types of microstructures:
1st rank laminate is grey; stronger isotropic material is black

The relaxed problem assumes the form

$$\min\left\{ f(w) \middle| w \quad \text{solves (2),(4),(5)}, m_2 \in L^\infty(\Omega;[0,1]), \int_\Omega m_2 dx = C_2 \right\} \tag{6}$$

The theory of the problem of minimal compliance of two-material plates teaches us that the 2nd rank microstructures, as shown in Fig.2, suffice to achieve the exact result. However, it can be proved that in the case considered one can confine the design to the 1st rank laminated microstructure of the core. Thus the set $G_{m_2}^{per}$ refers to 1st rank laminates, characterized by the design variables ω_2, ϕ, with $\theta_2 = 0$. The effective moduli $\left(H^{\alpha\beta}\right)$ are determined by the formula of Tartar, see Sec. 5.6.4 in Lewiński and Telega [5], applied iteratively. The equilibrium problem is solved with using the DSG finite element, see Bletzinger et al.[2], free of shear locking and satisfying the convergence consistency criteria of Strang. The optimization has been performed with using the COC method and the updating schemes reported in Bendsøe [1]. A similar algorithm has been previously applied to thin plates in Czarnecki and Lewiński [3]. The optimal inclination of ribs is given by the formula

$$\hat{\phi} = \chi - \chi_0, \quad \chi_0 = \frac{1}{2}\arctan\left(\frac{2H_0^{12}}{H_0^{11} - H_0^{22}}\right) \tag{7}$$

where $\chi = \angle(\mathbf{e}_1, \boldsymbol{\gamma})$ and $\left(H_0^{\alpha\beta}\right)$ refer to the basis $\left(\mathbf{e}_1^0, \mathbf{e}_2^0\right)$, see Fig.2. Thus we find the optimal distributions of m_2 in the core layer, see Fig.3. The optimal layout of the core is composed of the weaker material, of the stronger material and of laminates of 1st rank.

The layouts in Figs.3,4 refer to the square plate (AB=5.0 m., c = 16 cm, d = 0.3 cm,) clamped along the sides AB, CD and supported along BC, DA such that there $w = 0, \varphi_2 = 0$. The plate is subjected to a uniform transverse loading. The data are specifically chosen. The faces are made of an isotropic material of moduli E=210.0 GPa, ν=0.21. The core layer is made of two kinds of regular honeycombs of effective transverse shear moduli assessed here by the known formula $\mu_\gamma = 0.577\, t\, G_\gamma / l$; t = 0.02 cm represents the thickness of the honeycomb wall, while l = 1.00 cm is a length of the side of a honeycomb cell and G_1 = 15.22 GPa, G_2 =26.58 GPa are shear moduli of two materials of the honeycomb. The shear stiffnesses of the plate are computed by $H_\gamma^{\lambda\mu} = H_\gamma \delta^{\lambda\mu}$, $H_\gamma = (2b^2/c)\mu_\gamma$, b=c+d/2, see Sec.6.3 of [5], where the details of computing the bending stiffnesses can be found. The stronger material occupies 0.51 part of the domain Ω. The computations have been performed with using the updating scheme, see [1], with the damping factor $\eta = 0.75$ and the move limit $\zeta = 0.015$. The optimal layouts of the area fraction of both the materials compare favorably with 3D layouts found within the relevant relaxed formulation by homogenization, for various boundary conditions.

References

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