RADIAL-TYPE APPROXIMATION TECHNIQUE FOR A SPACE-TIME MULTISCALE COMPUTATIONAL STRATEGY

Anthony Nouy*, Pierre Ladevèze*

LMT-Cachan (ENS Cachan / CNRS / University Paris 6)
61, avenue du Président Wilson, F-94235 Cachan CEDEX, France

Summary A new multiscale computational strategy was introduced recently for the analysis of structures. It involves an automatic homogenization procedure in space and in time. This procedure requires the resolution of a large number of linear evolution equations. Here, we present a robust approximate resolution technique for these equations, based on the concept of radial approximation. It leads to the construction of a relevant reduced basis of space functions and it is particularly suitable for the analysis of composite structures.

INTRODUCTION

The behavior of heterogeneous structures, such as composite structures, often results from complex phenomena taking place on the microscale. Attempts to predict the behavior of such structures have led to the development of increasingly sophisticated models describing the material on a very fine scale compared to the scale of the structure. Therefore, the numerical simulation of these problems requires the development of efficient computational strategies taking into account these multiscale aspects. For the description of local phenomena with short lengths of variation in space as well as in time to be accurate, the calculations must be performed on a fine discretization of the structure and of the time interval being considered. This condition leads to problems with very large numbers of degrees of freedom, whose computation costs with standard calculation codes are generally prohibitive. One of the main objectives in the last few decades has consisted in developing efficient and robust computational strategies suitable for this type of problem.

A new multiscale computational strategy for nonlinear evolution problems was introduced recently. This strategy involves an automatic homogenization technique in space as well as in time [3, 4] which is an extension of previous works limited to space [2]. This strategy, developed in a general framework, makes no *a priori* assumption on the form of the solution and, therefore, does not have the limitations of standard homogenization techniques. Until now, it has been developed in the framework of small disturbances of (visco)plastic structures under possible contact conditions with or without friction.

THE MULTISCALE COMPUTATIONAL STRATEGY

The first characteristic of this strategy consists in partitioning the structure into substructures and interfaces, each of these entities having its own variables and equations. The change of scale takes place only at the interfaces, where forces and displacements are split into "macro" and "micro" contributions. The macro quantities have a real physical meaning: they are some mean values of the forces and displacements over space and time. The second characteristic of this strategy is the use of the LATIN method [1], an iterative resolution technique taking into consideration the whole time interval being studied. At each iteration, one must solve a homogenized macro problem defined over the whole space-time domain, and a set of independent micro problems, which are linear evolution problems defined within each substructure, at their boundaries, and over the time interval being studied.

RADIAL APPROXIMATION TECHNIQUE

The resolution of a large number of micro problems at each iteration can lead to prohibitive calculation costs if classical incremental techniques are used. Moreover, when dealing with composite structures containing a large number of similar substructures, the operators and right-hand sides of the micro problems may vary only slightly from one resolution to the next, but classical techniques are unable to take advantage of this characteristic.

A possible answer to this problem is an alternative resolution technique based on the concept of generalized radial approximation [1]. This technique, which relies on strong mechanical bases and is often used in the context of the LATIN method, consists in approximating a solution $u(\underline{x}, t)$ defined over the space-time domain by a sum of radial functions, each of which is the product of a scalar function of the time variable by a function of the space variable, i.e.

$$u(\underline{x}, t) = \sum_i \lambda^i(t) U^i(\underline{x})$$

Both space and time functions are *a priori* unknown. First of all, this approach reduces the computation cost drastically by transforming an evolution problem defined over the space-time domain into a series of uncoupled problems defined either over the space domain or over the time domain. It also reduces the storage cost for the variables defined over the space-time domain. Last but not least, it enables the construction of a relevant reduced basis of space functions which turns out to be particularly suitable for composite structures containing families of many identical substructures.

Of course, the radial approximation is not defined uniquely. The classical approach is to define this approximation using an orthogonality criterion associated with a primal or dual formulation of the evolution equation [1]. This definition leads

to the "best" approximation only for particular cases, and it provides no simple and systematic way to evaluate the quality of this approximation. Still, this classical approach is efficient in the single-scale framework. However, in some special situations, it may lead to non-convergence of the LATIN method. This lack of robustness is particularly notable for the multiscale computational strategy.

Here, we propose a new and extremely robust approach which maintains a completely mixed vision of the evolution problem. In fact, the search directions of the proposed iterative strategy can be interpreted as linearized constitutive relations. A natural and mechanics-based approach then consists in formulating the micro problem as the minimization of the constitutive relation error associated with the search directions. A radial approximation is searched for all the variables on a substructure, static and kinematic. The definition of the best radial functions is simply that they minimize the constitutive relation error. With this new approach, the approximation relates only to the search directions, which are the parameters of the strategy. Compared to the classical method, this approach certainly increases the computational cost, but it also improves the effectiveness and the robustness of the strategy. Moreover, it now provides a simple and relevant error criterion to evaluate the quality of the radial approximation.

The construction of the space functions is by far the most expensive step. Therefore, it can be interesting to store and reuse these functions. The space functions constructed at each iteration can be reused systematically during the subsequent iterations. Moreover, in the case of composite structures, the basis of the space functions is shared by a large number of identical substructures. Therefore, the initialization step of a micro problem on a particular substructure can reuse the space functions defined in other substructures during the previous iterations (or even the current iteration). Then, the resolution of a micro problem begins with an initialization step which consists in seeking a set of time functions associated with an already existing basis of space functions. The operators and the right-hand sides of the micro problems vary little from one iteration to the next, and even from one substructure to another. Therefore, this initialization step is often sufficient to obtain a good approximation of the solution of the micro problem.

Figure 1 shows a 2D problem of a composite structure containing cracks (unilateral contact with friction). The materials are viscoelastic. Figure 2 shows that very few new sets of radial functions were built at each iteration. Moreover, this number decreases as the iterations progress.
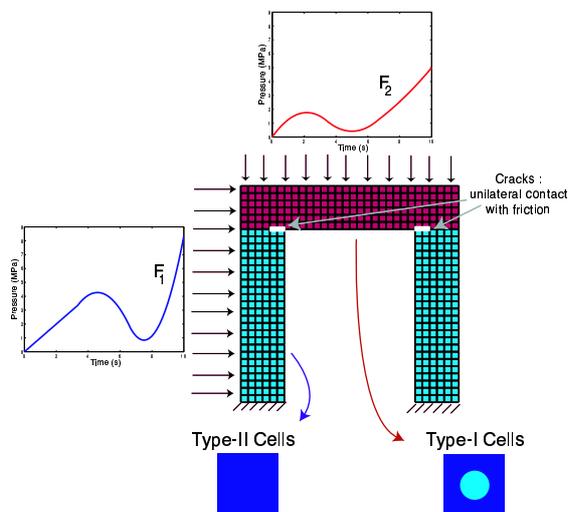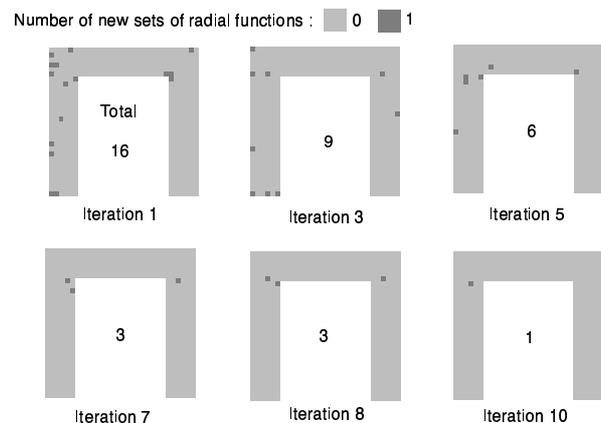

Figure 1. Description of the problem

Figure 2. Number of new sets of radial functions per iteration and per substructure

CONCLUSION

The presented approximation technique leads to significant savings in terms of computational and storage costs without affecting the efficiency and the robustness of the multiscale computational strategy. It could also lead to the construction of pertinent reduced models for heterogeneous structures and allow analysis such as parametric analysis, optimizations, which nowadays can not be carried out with experiments or computational techniques.

References

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