NEW NUMERICAL METHOD FOR COMPLEX INTERACTING FLOWS

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Summary New numerical method of second order accuracy for solving the boundary layer equation with interaction is suggested. Flow past a weak corner of a body contour was computed first. Second solved problem with using of proposed method is a mixing of boundary layers flowing from the edge of a wing.

New numerical method of second order accuracy for solving the boundary layer equation with interaction is suggested. It is well known that given displacement thickness defines solution of 2D boundary layer equation by the only way. Owing to this fact iterative methods were constructed to solve interactive problems. Taken first displacement thickness one can solve BL equation and find corresponding pressure gradient. Using computed pressure gradient new displacement thickness may be calculated through interaction law. Repeat of such steps gives a procedure to find the solution of the posted interacting problem. But convergence of such methods is not too high and depends essentially on the complexity of studied flow. Next important step in development of numerical methods was a so-called direct one, see Sychev *et al.* (1998) and references therein. That is equations at all points of grid are considered as algebraic equations which define corresponding unknown variables of flow field. Then this nonlinear algebraic system of equations is solved using Newton method. Advantage of this method is explicit form of Jacobian matrix which is computed at each iteration and gives high convergence. Shortcoming is too great amount of variables and as a result too big linear system to be solved at each iteration. Usually exact treating of such big linear system is possible using special measures. In spite of this shortcoming direct method is the most effective one at the time at least for 2D flows.

Proposed in this paper method is addressed to combine advantages of ones mentioned above. Displacement thickness function is considered as the only unknown grid function in the problem. As a result amount of unknowns equals to number of nodes along X-axis of grid. The main problem now is to create a set of equations which define this displacement thickness function. Such set of equations may be formulated as follows.
Interaction law taken from outer inviscid region of flow usually means linear relationship between displacement thickness function $A_i(x_i)$ and induced pressure gradient $\left.\frac{\partial p}{\partial x}\right|_{inv}$. From other side function $A_i(x_i)$ defines solution of BL equation and as a result gives $\left.\frac{\partial p}{\partial x}\right|_{v}$ unknown in advance. It is clear that difference between above pressure gradients must be zero if $A(x)$ is a solution of posted problem. As a result we can formulate set of implicit equations:

$$F_i = \left.\frac{\partial p}{\partial x}\right|_{inv} - \left.\frac{\partial p}{\partial x}\right|_{v} = 0 \qquad i = 1,..., NX.$$

High convergence is based on Newton method of solving the implicit nonlinear system of equations. Unlike the direct method, Jacobian matrix in this case must be computed numerically with aid of a special procedure. Initial system of BL equation was linearized at each iteration on basis of current solution. Solving of linear system for unity perturbations of grid function $A_i(x_i)$ at each node i gives perturbations of pressure gradient and as a result Jacobian matrix. Interesting advantage is found that computation of Jacobian matrix may be made by parallel processors simultaneously. It is important to say that given method may be very effective for 3D flows in comparison with direct one in which number of algebraic equations becomes enormous.

Flow past a weak corner of a body contour was computed as a test. This problem was posted by Stewartson (1970). The results are displayed in figure (a) the growing of length of separation region l against α values and (b) streamlines of the solution for $\alpha = -4.67$ for large separation zone. Method allowed to derive two branches of the solution (a).

Method was used for complex flow calculation namely for mixing of two boundary layers with interaction which flow past lateral or trailing edge of a wing. The wing was replaced by a flat plate. The interaction problem is similar to symmetrical one near a trailing edge of a flat plate which was formulated by Stewartson (1969) and Messiter (1970). In the considered case the flow is not symmetrical one and contains two parameters $\frac{\lambda_+}{\lambda_-}$ and $\left(\frac{u_+}{u_-}\right)^2$ which mean skin friction and velocity relations in the unperturbed free stream on the upper and lower side of the plate. Additional condition for this flow is equality of pressures in the wake computed from upper and bottom potential flows. To calculate such flow upper and bottom displacement functions were taken as independent variables. Upper pressure is found as a function of upper displacement thickness. Solution of a mixed problem of seeking of analytical function in lower half plane gives a pressure distribution on the lower side of plate. As a result pressure distribution on the surface is a linear function of both displacement thicknesses and implicit set of equations introduced earlier are applicable for the problem.

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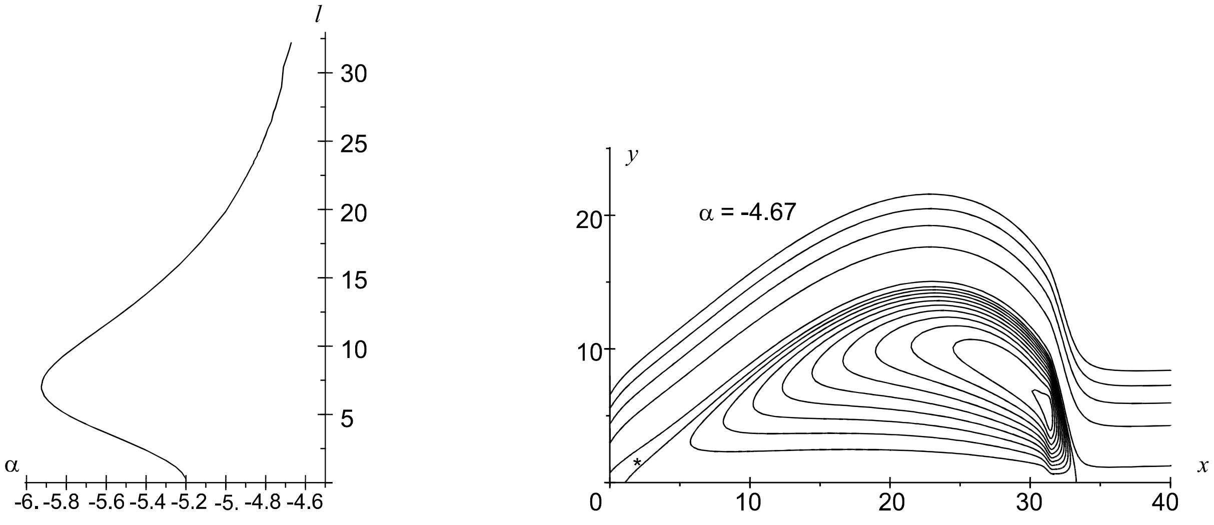

(a) (b)

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