INTERFACIAL JUMP CONDITIONS IN STRAIN-GRADIENT PLASTICITY AND RELATIONS OF HALL-PETCH TYPE

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EXTENDED ABSTRACT

Strain-gradient plasticity requires boundary conditions, and jump conditions across interfaces, additional to those required in ordinary plasticity. In recent work, Fleck and Willis [1] demonstrated an effect of the scale of the microstructure on the effective flow stress of a composite. The effective flow stress had a definite upper limit, equal to the "Taylor upper bound", for very fine microstructure; this was because plastic strain-gradients were obliged to tend to zero as the scale of the microstructure reduced giving, in the limit, the Taylor "uniform plastic strain" approximation. However, Fleck and Willis adopted the simplest assumption, that both plastic strain and higher-order traction were continuous across interfaces. This need not be so. For instance, a dislocation crossing an interface will leave a ledge on the interface and so add to its energy. In the work to be described, we retain the assumption that plastic strain is continuous across an interface but penalise dislocations crossing the interface by admitting an interfacial potential that is a function of the plastic strain; this induces a jump in the higher-order traction, which is analogous to allowing dislocations to pile up. The theory is illustrated by a number of simple one-dimensional examples, both linear and nonlinear. The microstructure is taken to be periodic and the "homogenised", or "effective", response is studied by taking the total strain to be periodic with prescribed mean value; the plastic strain is likewise periodic but its mean value is obtained from the solution. A pronounced scale-dependence of effective flow stress -- a kind of Hall-Petch relation -- is obtained in every case. It is due entirely to the presence of the interfacial potential: the original Fleck-Willis analysis would yield no scale-dependence in the one-dimensional cases that are considered.

One-dimensional problems with periodically distributed interfaces require only the solution of ordinary differential equations. It is also possible to evaluate the utility of an approximation scheme, of wider applicability, for solving nonlinear problems, by applying it in the nonlinear periodic setting. The scheme employs a linear “comparison problem”, in which the parameters are chosen to reproduce, as closely as possible, the field in the original nonlinear problem, in the way introduced for non-strain-gradient composites by Ponte Castaneda [2] and also employed by Fleck and Willis [1] in the strain-gradient context. Work in progress includes allowance for randomly-distributed interfaces, using this scheme of approximation.

Implementation of the present model in two or three dimensions is planned; it is likely to predict the attainment of effective flow stresses larger than the Taylor upper bound.

References

[1] N.A. Fleck and J.R. Willis, “Bounds and estimates for the effect of strain gradients upon the effective plastic properties of an isotropic two phase composite”. J. Mech. Phys. Solids (to appear).
[2] P. Ponte Castaneda, “The effective mechanical properties of nonlinear isotropic composites”, J. Mech. Phys. Solids **39,** 45-71, 1991.