PASSIVE VIBRATION CONTROL OF A PIECEWISE LINEAR BEAM SYSTEM

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Summary A linear Dynamic Vibration Absorber (DVA) is applied to suppress the vibrations in a harmonically excited piecewise linear beam system. Both an undamped and a damped DVA are considered. Results of experiments and simulations are presented and show good resemblance. The undamped DVA is able to suppress the first harmonic resonance peak. The damped DVA guarantees vibration reduction over a wider frequency range.

INTRODUCTION

Several reasons can be given for the need to reduce vibrations in a mechanical system: e.g. avoiding damage, increasing the lifetime of the system, increasing comfort for human beings, and decreasing sound radiation. A Dynamic Vibration Absorber (DVA) is a well-known cheap, simple, passive controller for neutralising the vibrations of the structure to which it is attached. It consists of a mass-spring(-damper) system which parameters are tuned to obtain maximal vibration reduction. The application of DVA's to linear systems has been investigated by many authors, see e.g. [5]. In the past piecewise linear systems with one degree of freedom (dof) were studied [6,7] inspired by the fact that in engineering practice many of such systems can be found. Examples are: 1) one-sided springs in folded solar array systems for space applications [4], 2) the periodic slackening of a mooring rope in off-shore applications [8] and 3) elastic stops in a pantograph carbon collector strip suspension [3]. In this paper a linear DVA will be applied to an archetype piecewise linear beam system to investigate if linear DVA's are capable of reducing vibrations in this type of systems, see also [1].

EXPERIMENTAL SET-UP

Figure 1 shows a schematic view of the experimental set-up. A steel beam is supported by two leaf springs at both ends. Below it a shorter clamped-clamped beam acts as a one-sided spring, which only is loaded if the middle of the main beam has downward deflection. Above the middle of the main beam a rotating mass unbalance realizes harmonic excitation. This unbalance is driven by a motor via a shaft with a flexible coupling. Left from the rotating mass unbalance the symmetric DVA is visible. It consists of two mass-leaf spring systems. The damped DVA is realised by addition of two dampers at the ends of the leaf springs.

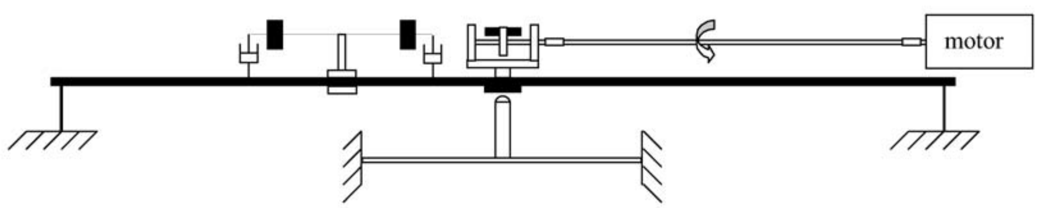

Figure 1 Schematic view of the experimental set-up

NUMERICAL MODEL

The linear part of the model without DVA is modelled using Euler beam elements resulting in a model with 111 dof. In order to decrease computational time while maintaining accuracy for the frequency range of interest a dynamic reduction method [2] based on free-interface eigenmodes and residual flexibility modes is applied to this model. The Ritz approximation of the displacement field consists of a linear combination of 3 free-interface eigenmodes (up to a cut-off frequency of 100 Hz) with corresponding generalised dof p_1, p_2, p_3 and 2 residual flexibility modes. The latter are defined for q_m, the transversal displacement of the middle of the main beam, and q_{ab}, the transversal displacement at the position where the DVA is attached. The equations of motion of the reduced model are given by:

$$M_R \ddot{p} + B_R \dot{p} + K_T p = f_R \qquad \text{where} \qquad K_T = \begin{cases} K_R & if \quad q_m > 0 \\ K_R + K_{nl} & if \quad q_m \leq 0 \end{cases}$$

The reduced displacement column is given by $p = [p_1 \quad p_2 \quad p_3 \quad q_m \quad q_{ab} \quad q_{am}]^T$, where q_{am} is the transversal displacement of the DVA (the DVA is modelled as a single dof mass-spring-damper system). Only the 4[th] element of the reduced external force column f_R is non-zero. It is a harmonic function with amplitude $mr\omega^2$ (m : mass unbalance, r : radius of mass unbalance, ω : radial rotational frequency) and excitation frequency $f = \omega / 2\pi$. The clamped-clamped beam is modelled as a mass-less one-sided spring causing piecewise linearity and resulting in stiffness matrix K_{nl} (the lowest eigenfrequency of this beam is high enough to justify this approximation). An important system parameter is the quotient of the stiffness of the one-sided spring and the stiffness associated with the first eigenmode, which is a measure for the amount of nonlinearity in the system. In this case this quotient is 4.6.

RESULTS

Figure 2 shows the maximum displacement of q_m for excitation frequencies between 0-60 Hz for 3 system configurations.

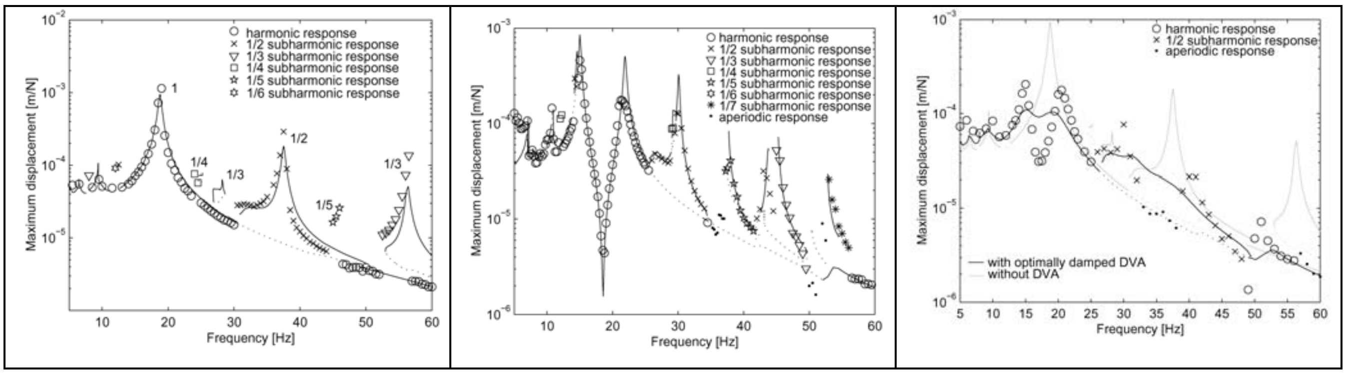

Figure 2 Maximum displacement without DVA (a), with undamped DVA (b), and with damped DVA (c) against excitation frequency. Symbols refer to experimental results, solid and dotted lines to resp. stable and unstable numerical periodic solutions.

Figure 2a: Piecewise linear beam system without DVA
Harmonic resonance occurs near 19 Hz. Related to this resonance ½ and 1/3 subharmonic resonances occur at resp. 38 Hz and 57 Hz and 2nd and 3rd superharmonic resonances at resp. 9.5 Hz and 6.5 Hz. The ¼ subharmonics near 26 Hz are related to the 3rd superharmonic resonance and the 1/3 and 1/5 subharmonics near 28 Hz and 47 Hz resp. to the 2nd superharmonic resonance.

Figure 2b: Piecewise linear beam system with undamped DVA
The eigenfrequency of the undamped DVA is tuned to 19 Hz, enabling the DVA to generate a counteracting force for the excitation force at this frequency. Indeed, at 19 Hz now an anti-resonance occurs. Instead, two new harmonic resonances occur near 15 Hz and 22 Hz. Related to these resonances ½ subharmonic resonances occur near resp. 30 Hz and 44 Hz. The other subharmonic and superharmonic resonances in figure 2b can also be identified. Finally, a quasi-periodic → locked → chaotic sequence is found near 35 Hz.

Figure 2c: Piecewise linear beam system with damped DVA
The optimally damped DVA reduces the vibrations over a wide frequency range. Next to the reduction of the harmonic and ½ subharmonic resonance, the 1/3 subharmonic resonance even completely disappears. Unfortunately, the dampers used in the experiment could not be given the optimal damping values. Nevertheless, the trend in the experiments is correct.

CONCLUSIONS

The undamped linear DVA is able to suppress the first harmonic resonance of the piecewise linear system. The damped DVA guarantees vibration reduction over a wider frequency range. Experimental and numerical solutions corresponded well.

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