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SIZE-EFFECTS IN VOID GROWTH

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Summary A recent finite strain generalization of a higher order strain gradient plasticity theory is used to study how void growth is influenced by the material length parameters on the micron scale. The analyzes are based on a unit cell model considering either a plane strain model with cylindrical voids or an axisymmetric model with more realistic arrays of spherical voids. It is shown that gradient contributions to the material hardening suppress void growth significantly when voids are small.

INTRODUCTION

Recent experimental and numerical work (e.g. [1]) have shown that void growth on the micron scale exhibit significant size-effects. These findings are supported by theoretical arguments based on the concept of geometrically necessary dislocations from dislocation mechanics. Such size effects cannot be modeled by conventional continuum theories, since they use no material length parameters. In recent years several gradient theories have been proposed, which can model size effects by using material length parameters. Some of these theories are of higher order nature, using higher order stresses as work conjugates to higher order strains and higher order boundary conditions, while others are of lower order nature, in that they retain the structure of conventional boundary value problems in solid mechanics.

While some studies of void growth are based on growth of a single void in an infinite medium [2], the present work studies size effects for void growth in elastic-plastic materials with periodic distributions of cylindrical or spherical voids. Since the focus is on size-effects, a recent higher order strain gradient plasticity theory proposed in [3] is used to model the matrix material. The problem of void growth inherently involves large plastic deformations, which means that a finite strain model is needed. Hence, a recent finite strain generalization proposed in [4] is implemented into a finite element program and used to study the problem numerically.

The results presented in the present work focus on the effect of material length parameters on the overall properties of metals containing voids. Results are presented which highlight the strength of the materials containing different size voids, and which illustrate the growth in void volume vs. overall deformation at different ratios of the void radius to the material length scales.

FINITE STRAIN GRADIENT PLASTICITY MODEL

The finite strain generalization [4] of the strain gradient plasticity theory proposed in [3] is based on the following form of the principle of virtual work in the current configuration

$$\int_V \left(\sigma_{ij}\delta\dot\epsilon_{ij} + \left(Q - \sigma_{(e)}\right)\delta\dot\epsilon^P + \tau_i\delta\dot\epsilon^P_{,i}\right) dV = \int_S \left(T_i\delta\dot u_i + t\delta\dot\epsilon^P\right) dS \tag{1}$$

Here, σ_{ij} is the conventional Cauchy stress tensor, $\dot\epsilon_{ij}$ is the strain rate, $\sigma_{(e)}$ is von Mises's effective stress, Q is a generalized effective stress, and $\dot\epsilon^P = \sqrt{\frac{2}{3}\dot\epsilon^P_{ij}\dot\epsilon^P_{ij}}$ is the effective plastic strain rate, with $\dot\epsilon^P_{ij}$ denoting the plastic strain rate. The higher order stress, which is work conjugate to the gradient of the effective plastic strain, is denoted τ_i. With N_i denoting the unit surface normal, the right hand side of the principle of virtual work consists of contributions from the traction $T_i = \sigma_{ij}N_j$ which works against the displacements u_i, as well as from the higher order traction $t = \tau_i N_i$, which work against the effective plastic strain.

Letting J denote the material dilatation, Kirchhoff stress measures are defined as

$$\varsigma_{ij} = J\sigma_{ij}, \qquad \sigma^{\varsigma}_{(e)} = J\sigma_{(e)}, \qquad \rho_i = J\tau_i, \qquad q = JQ \tag{2}$$

With an updated Lagrangian formulation in mind, these Kirchhoff stresses can be used to rewrite the incremental principle of virtual work as follows

$$\int_V \left(\overset{\triangledown}{\varsigma}_{ij}\,\delta\dot\epsilon_{ij} - \sigma_{ij}\left(2\dot\epsilon_{ik}\delta\dot\epsilon_{kj} - \dot e_{kj}\delta\dot e_{ki}\right) + \left(\dot q - \dot\sigma^{\varsigma}_{(e)}\right)\delta\dot\epsilon^P + \overset{\vee}{\rho}_i\,\delta\dot\epsilon^P_{0,i}\right) dV = \int_S \left(\dot T_{0i}\delta\dot u_i + \dot t_0\delta\dot\epsilon^P\right) dS \tag{3}$$

Here, derivatives are evaluated in the reference configuration, and the right hand side is based on nominal tractions. The constitutive relation for the Jaumann rate of the Kirchhoff stress is taken to be

$$\overset{\triangledown}{\varsigma}_{ij} = \mathcal{R}_{ijkl}(\dot\epsilon_{kl} - \dot\epsilon^P_{kl}) \tag{4}$$

where $\mathcal{R}_{ijkl}$ is the isotropic elastic stiffness tensor. Denoting the hardening modulus by h, and defining gradient tensors A_{ij}, B_i, and C, as in [3], the constitutive relation for the generalized effective stress can be expressed as

$$\dot q = h\left(\dot\epsilon^P + \frac{1}{2}B_i\dot\epsilon^P_{,i} + C\dot\epsilon^P\right) \tag{5}$$

and the constitutive equation for the convected derivative of the higher order Kirchhoff stress as

$$\overset{\vee}{\rho}_i = h(A_{ij}\dot{\epsilon}^P_{,j} + \frac{1}{2}B_i\dot{\epsilon}^P) \tag{6}$$

NUMERICAL RESULTS AND DISCUSSION

The finite strain gradient plasticity model has been incorporated in an updated Lagrangian finite element program, using 8-noded isoparametric elements. Due to the higher order nature of the theory, the discretized equations are based on plastic strain degrees of freedom on equal footing with displacement degrees of freedom.

Figure 1 shows overall results for a simple cubic distribution of cylindrical voids under plane strain deformation. Figure 1a shows the nominal stress, σ_n, as a function of the linear strain, ϵ_l, while Figure 1b shows the ratio of current void volume to initial void volume, V/V_0. Each family of curves corresponds to a certain ratio of void radius to half the void distance, $\frac{R_0}{a_0}$, with varying values of the internal material length parameter l_*. The solid curve shows results for a material without voids, the dashed curves show results for a material with $\frac{R_0}{a_0} = 0.25$, and the dotted curves show results for a material with $\frac{R_0}{a_0} = 0.5$. The markings on the curves in Figure 1a show the maximum load point. The figure illustrates how the strength decreases with increasing void volume fraction, but more interestingly it shows how the strength increases with increasing material length parameter. From Figure 1b it is seen how the increase in strength with the material length parameter is linked to the suppression of void growth, which is due to the gradient hardening.

(a) (b)

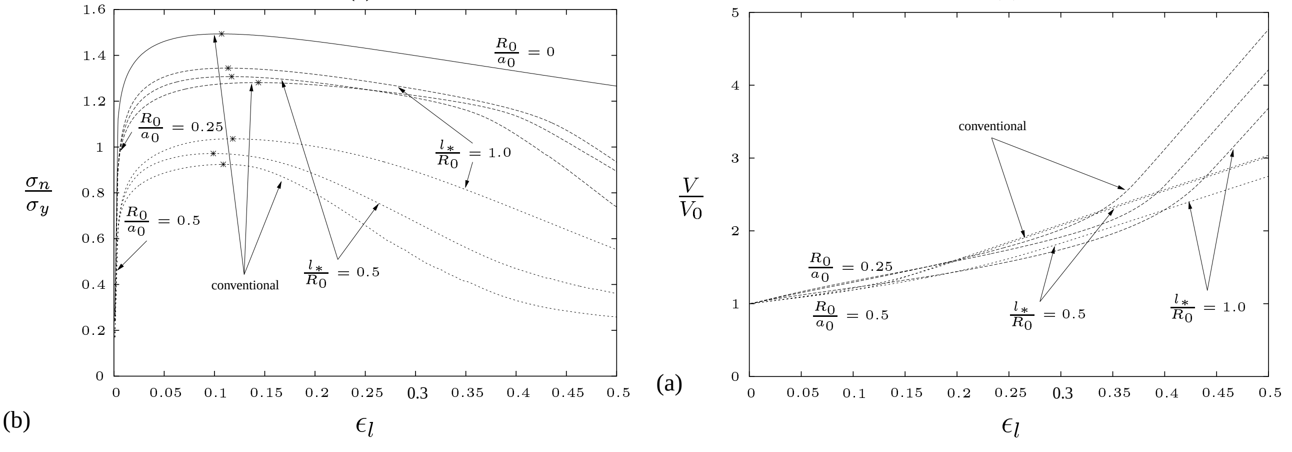

Figure 1. The figures show overall results for materials with different size voids and different internal material length parameters. Figure (a) shows the nominal stress as a function of the overall linear strain, and figure (b) shows the void volume normalized with the initial void volume as a function of the overall linear strain.

Thus the work shows how a strain gradient plasticity theory can be used to model the fact that small voids grow less than larger voids. Due to the lack of length parameters in conventional theories they cannot capture such size-effects. It is illustrated how gradient hardening increases the strength of an elastic-plastic material with voids and suppresses void growth when compared to conventional predictions. The results in Figure 1 are based on plane strain analyzes of cylindrical voids, but the work also includes axisymmetric analyzes for the growth of initially spherical voids in elastic-plastic solids. The results also highlight the effect of triaxial loading which has a significant effect on void growth.

References

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