AN ENERGY CONSERVING SCHEME FOR TIME DEPENDENT PROBLEMS USING THE EXTENDED FINITE ELEMENT METHOD

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Summary Lots of techniques have been developed to take into account discontinuities such as cracks, holes or material interfaces. The eXtended Finite Element Method (X-FEM) for example has first been presented for crack modeling [1] but is also used for arbitrary discontinuities. This paper proposes, with a proof of stability for the numerical scheme in the linear case, a generalization of X-FEM for time dependent problems with an application to dynamic crack propagation.

NUMERICAL STABILITY AND ENRICHMENT STRATEGY

In this paper, we propose a study of Newmark type schemes for problem with evolving discretization. This study, applied for the particular case of dynamic crack growth simulation using XFEM, allowed us to define an enrichement strategy. This strategy is satisfying stabilty conditions and is also energy preserving.

$$[B_\alpha] = \left[\sqrt{r}sin(\frac{\theta}{2}), \sqrt{r}cos(\frac{\theta}{2}), \sqrt{r}sin(\frac{\theta}{2})sin(\theta), \sqrt{r}cos(\frac{\theta}{2})sin(\theta)\right] \quad (1)$$

An enrichment is added to the displacement, velocity and acceleration fields with the same basis of enriched shape functions (discontinuous step function, and singular functions listed in Eq. (1)) using the Partition of Unity Method [2](see Ref.[3] for an other approach). The enrichment strategy for time dependent problems is presented in Fig. 1.

Using X-FEM, new enriched shape functions must be considered to simulate the crack growth. All enrichments are kept from time t_n to t_{n+1} and the initialization of new degrees of freedom is made to zero (Eq. (2)).

This strategy is justified by a theoretical study of Newmark type schemes for problems with evolving discretization. This study is an extension of the energy method from Hughes and Belytschko [4] to dynamic problems with evolving discretization. This approach, called the balance recovery method [5], ensures stability and energy conservation. Using this approach, the discretization can evolve when the crack propagates without any instability or uncontroled energy transfert.

$$[U_n^{n+1}] = \begin{bmatrix} U_n^n \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2)$$

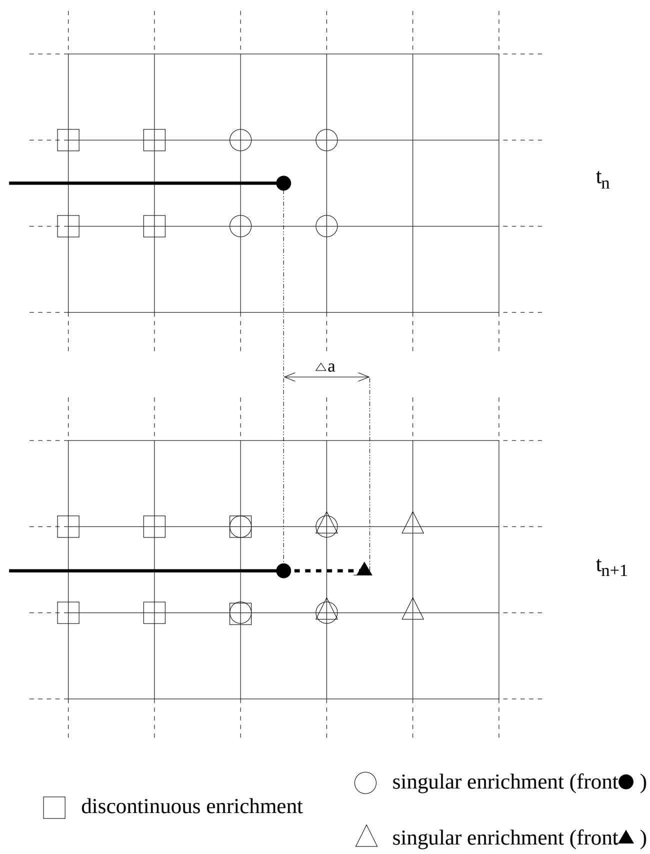

Figure 1. Enrichment strategy

EXAMPLE

To show the capabilities of the method, we calculate the dynamic propagation of a crack in a plate under prescribed displacement (see Fig.3).

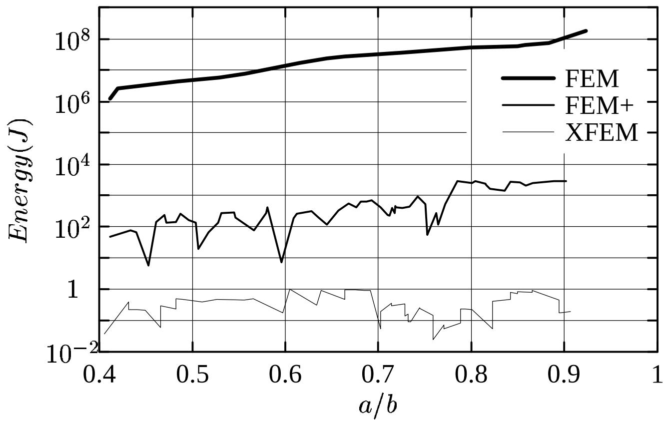

Figure 2. Cumulated energy out of balance

This example has already been calculated by authors in a classical FEM framework [5]. Using FEM, the balance recovery method (FEM+) reduces the cumulated energy out of balance but projections and remeshing needed in this framework still produce uncontroled energy transferts. With X-FEM, the presented enrichment strategy enables us to calculate the crack propagation satisfying energy conservation (see Fig. 2).

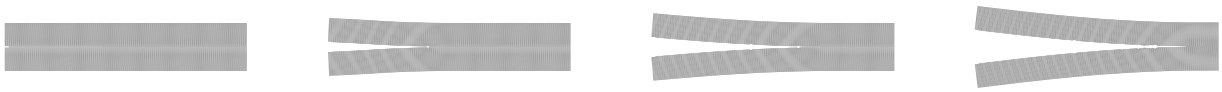

Figure 3. Dynamic crack growth in a plate under prescribed displacement

CONCLUSION

An enrichment strategy is developed to satisfy conditions obtained from a theoretical study. This strategy is energy preserving and satisfies the same stability conditions as Newmark type schemes. Numerical results reveal the advantage of X-FEM which avoids numerical problems due to remeshing and projections. This framework is still valid for other time dependent problems and this general enrichment strategy should be applicated to arbitrary moving discontinuities.

References

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