THE MIXING LAYER INSTABILITY OF WIND OVER A FLEXIBLE CROP CANOPY

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Summary A coupled fluid-stucture model is proposed to study the dynamics of a flexible crop canopy exposed to wind. The canopy is represented by an elastic continuous medium and coupled to the wind mixing layer through a drag load. The mixing layer instability is shown to remain the principle instability mechanism but its characteristics are modified when taking into account the flexible canopy. The size of the coherent structures is decreased as well as the instability growth rate.

INTRODUCTION

Lodging of crops and thigmomorphogenesis, which is the effect of wind on plants growth, have brought a large interest on the study of wind-induced plant motions [1, 2, 3]. Wind flow over a vegetal canopy is turbulent and leads to oscillatory plant motions. The mean wind velocity profile is known to be inflected at the top of the canopy, as a result of the momentum absorption by drag on the leaves [2]. The induced shear is at the origin of an instability mechanism similar to that of a mixing layer. This yields the formation of large scale coherent flow structures propagating over the canopy [1, 2]. Previous studies of wind-induced plant motions have represented the canopy as a set of mechanical oscillators [3, 4], with elastic interactions [5], and have considered simple wind load models. We propose a fully coupled fluid-structure model to study the dynamics of a flexible canopy exposed to wind. A mixing layer configuration with a broken line flow profile represents the wind flow over and inside the canopy. The flexible canopy is modeled by a wave equation following Doaré *et al* [5]. The fluid-structure coupling is realized through a drag load. We identify the main instability mechanism of the coupled problem, and study the effect of the mechanical characteristics of the canopy on the instability properties in the particular case of an alfalfa field.

THE MODEL

The vegetal canopy is composed of an infinite row of identical plants with elastic contacts, Figure 1. Following Doaré *et al* [5], it is modeled as an equivalent continuous medium, with mass m, flexion stiffness r and contact stiffness a . The position of any point of the canopy is described by the horizontal displacement $\underline{X}(x,y,t) = \chi(y)\ Q(x,t)\ \underline{e_x}$, χ being a mode shape, and Q the corresponding generalized displacement. The canopy movement results from a local drag force, acting on the equivalent surface of each plant and dependent on the difference between the local wind velocity and the horizontal plant velocity. This drag force is then projected on the modal shape χ. The dynamics of the canopy is thus governed by the following wave equation:

$$m\,\partial_{tt}Q \,+\, r\,Q \,-\, a\,\partial_{xx}Q \,=\, \int_0^h \frac{1}{2}\rho C D \left[\left(\underline{U} - \underline{\dot{X}}\right).\underline{e}_x\right]^2 \chi\, dy \qquad (1)$$

where C is a drag coefficient and D an effective diameter of a plant.

The wind velocity $\underline{U}$ is governed by the Euler equations within and above the canopy. A source term corresponding to the drag effect is added to the momentum equation. We neglect here the effect of fluid viscosity since the mixing layer instability mechanism is known to be inviscid [6].

In order to investigate the stability of a given basic velocity profile U_b and canopy displacement Q_b, we consider the small perturbations u, v, p and q. We use here a broken line profile U_b defined by a vorticity thickness δ and a shear parameter $R = (U_1 - U_2)/2U$ with the mean velocity $U = (U_1 + U_2)/2$ (see Figure 1). The momentum and mass balance for the flow and the wave equation for the canopy may be developed at the first order in terms of the perturbations which yields the set of coupled equations:

$$\begin{aligned}
\rho\,(\,\partial_t u \,+\, U_b\,\partial_x u \,+\, u\,\partial_y U_b\,) &= -\,\partial_x p - \rho C U_2 D\,/l^2\ \left(u - \chi\,\partial_t q\right) && (2)\\
\rho\,(\,\partial_t v \,+\, U_b\,\partial_x v\,) &= -\,\partial_y p && (3)\\
\nabla.\underline{u} &= 0 && (4)\\
m\,\partial_{tt}q \,+\, r\,q \,-\, a\,\partial_{xx}q &= \int_0^h \rho C U_2 D \left(u - \chi\,\partial_t q\right)\chi\, dy && (5)
\end{aligned}$$

The drag coefficient C in (2) is set to zero outside the canopy ($y > h$). We use here a linear mode shape $\chi(y) = y/h$. A traveling wave solution is sought in the form: $(u, v, p, q) = (\hat{u}, \hat{v}, \hat{p}, \hat{q})\ e^{i(kx - \omega t)}$. Assuming the flow to be irrotational, the analytical dispersion relation is obtained.

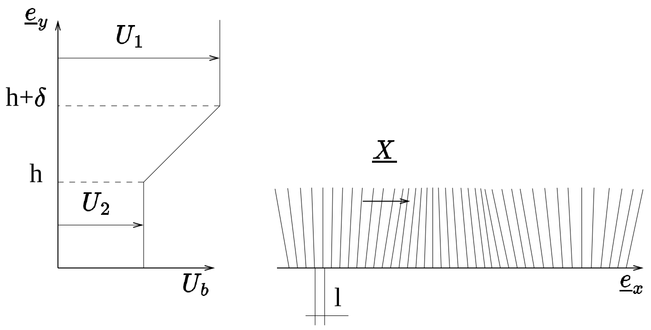

Figure 1. Basic flow U_b and model of the crop canopy.

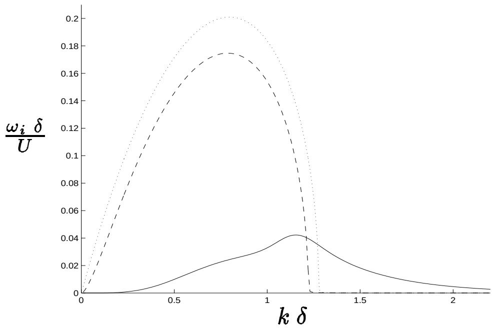

Figure 2. Dimensionless temporal branch. (—) model with experimental values of the parameters, (- -) $C = 0$: Kelvin-Helmholtz branch in a bounded medium, (· · ·) Kelvin-Helmholtz branch in an infinite medium.

ANALYSIS AND RESULTS

We analyze the temporal stability of propagating waves by calculating numerically the growth rate $\omega_i = Im(\omega)$ associated with a given real wavenumber k [6] through the dispersion relation. Only the most unstable branch $\omega_i = f(k)$ is considered. The temporal analysis is performed using experimental values for the parameters of the model. These are taken from experiments on alfalfa [5] and wind characteristics over a crop canopy [1]. We use here : $m/(\rho\delta^3) = 0.007$, $r/(\rho\delta U^2) = 0.009$, $a/(\rho\delta^3 U^2) = 1.5\ 10^{-5}$, $h/\delta = 1$, $l/\delta = 0.1$, $D/\delta = 0.02$, $C = 1$, $R = 0.5$. Figure 2 shows the temporal branch derived from the dispersion relation.

The growth rate may be compared with that arising from uncoupled fluid and solid dynamics by setting the drag coefficient C to zero. In that case, the unstable branch is that of the Kelvin-Helmholtz instability in a bounded domain, Figure 2. Then, by setting h/δ equal to infinity, the Kelvin-Helmholtz temporal branch for a broken-line profile in an infinite medium is recovered, with the most amplified wavenumber at $k\ \delta = 0.8$ [6]. We may therefore assert that taking into consideration the flexible canopy through the drag term modifies the shape of the Kelvin-Helmholtz temporal branch, but this latter is still the most amplified. The mixing layer instability therefore remains the main mechanism of vortex formation over canopies, even when considering the drag on the flexible plants. The first effect of the coupling by a drag force is to decrease the maximum growth rate of the instability : dissipative effects in mixing layers are indeed known to be stabilizing, see for instance [7].

The effect of the mechanical characteristics of the plants on the instability mechanism may be investigated by comparing the most amplified temporal branch between a flexible canopy and a rigid one. This results in an evolution of the most amplified wavenumber as a function of the dimensionless flexural compliance $s^* = \rho U^2 \delta / r$. The crop compliance has a non regular but significant effect on the size of the coherent stuctures arising from the instability. For the particular case of an alfalfa canopy using a $3\ m/s$ mean wind ($s^* \simeq 100$), the wavelength is approximatively 25 % smaller than it would be for an equivalent rigid canopy.

The mixing layer instability is known to be responsible for the existence of strong coherent motions of canopies exposed to wind [2]. We have solved here the interaction problem where the mixing layer dynamics is fully coupled with the motion of the canopy. The main conclusion is that for realistic values of the crop flexibility the mixing layer instability persists but its characteristics are significantly modified. The most amplified wavelength is reduced, as well as the corresponding growth rate of the instability. This may explain some discrepancies between the original mixing layer model and measurements of the size of coherent structures observed in wind over various canopies such as corn or forest as reported in [1].

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