DETERMINISTIC SIZE EFFECT IN THE STRENGTH OF CRACKED QUASI-BRITTLE STRUCTURES

B. L. Karihaloo, H. M. Abdalla, Q. Z. Xiao

School of Engineering, Cardiff University, Queen's Buildings, P O Box 925, Cardiff, CF24 0YF, UK

There are two fundamental causes of size effect in the strength of structures made from quasi-brittle materials, such as concrete, namely the material heterogeneity and the occurrence of discontinuities in the flow of stress, such as at cracks and notches. The former leads to the statistical size effect and the latter to the so-called deterministic size effect. In these materials, any crack or notch tips are blunted by the formation of a fracture process zone (FPZ) so that the deterministic strength size effect is less severe than that predicted by the Griffith theory for brittle materials. In the FPZ the stresses are redistributed and energy is dissipated which is thus not available for crack propagation. The size of FPZ can be commensurate with that of most structural elements. Only in very large structures can this size be regarded as small in comparison with their characteristic dimensions.

The redistribution of stresses and dissipation of energy in the FPZ were considered by Bažant who derived a size effect formula by performing a Taylor's expansion of the size of FPZ from its small asymptotic limit for large structures. This formula is claimed to be applicable to structures in the size range 1:32 but the determination of accurate coefficients appearing in it from tests on laboratory size specimens is questionable and has led to controversy in the literature.

Another approach to capturing the FPZ within a nonlinear theory of fracture for quasi-brittle materials is the so-called ficticious crack model (FCM) which has its origin in the Barenblatt-Dugdale cohesive zone concept. In this model, the FPZ ahead of a real crack is replaced with a fictitious crack in which the material exhibits softening with a residual stress transfer capability across the crack faces dependent on the crack opening displacement, $\sigma(w)$. The faces of the fictitious crack are assumed to close smoothly, so that the stress is finite at its tip and the stress intensity factor (SIF) due to external loading is cancelled by the closure pressure $\sigma(w)$ in the FPZ. The real crack grows only when $w = w_c$. The area under the $\sigma(w)$ diagram from $0 \le w \le w_c$ is the true specific fracture energy G_F of the material. This approach was taken by Karihaloo who however made several approximations which rendered the predictions of his size effect formula suspect for small structures. In the derivation of this formula, it was recognized that quasi-brittle materials develop a diffuse FPZ before the formation of a traction-free crack whose size can be commensurate with that of a small test specimen. Within the FPZ the stresses are redistributed so that it is necessary to consider not only the singular term in the asymptotic crack tip field but also higher order non-singular terms. In the derivation however approximations were used for these higher order terms. Moreover, weight functions for a semi-infinite crack in an infinite plane were used instead of the correct weight functions for a finite crack in a finite specimen.

In this talk, we shall present results in which all these approximations have been eliminated and a new size effect formula derived. This formula is found to be in excellent agreement with test data and to cover a very large size range of 1:80. Its predictions are confirmed by experimental observations, namely that the deterministic strength size effect becomes stronger as the size of structure increases but weakens as the size of the crack relative to the structural size reduces.

The mathematical formulation is based on the decomposition of a traction-free crack of size a in a structure of characteristic size W with a FPZ of length ℓ_p at its tip into (i) a traction-free crack with the following stress field at its tip

$$\sigma_y(r) = \frac{a_1}{\sqrt{r}} + 3a_3\sqrt{r} + 5a_5 r^{\frac{3}{2}} + \cdots \tag{1}$$

and (ii) a FPZ with the net stress $[\sigma(s) - \sigma_0(\ell_p - s)]$ and the displacement $w(s)$ across its faces. The coefficients a_1, a_3, a_5, … depend on a, W, applied load σ and shape of the body. The authors have obtained these coefficients for several commonly used specimen geometries. Note that a_1 =

$K_I/\sqrt{(2\pi)}$ where K_I is the applied mode I SIF. The opening displacement of the fictitious crack faces $w(t)$ (representing the FPZ) are given by

$$\int_0^{l_p} g(s,t;a)\left[\sigma(s)-\sigma_0\left(l_p-s\right)\right]ds = -w(t) \tag{2}$$

and the smooth closure condition for the fictitious crack tip is

$$\int_0^{l_p} k(s;a)\left[\sigma(s)-\sigma_0\left(l_p-s\right)\right]ds = 0 \tag{3}$$

The weight functions $g(s, t; a)$ and $k(s; a)$ are the respective crack face opening displacement at the location t and the SIF at the finite crack tip in a specimen of finite size due to a pair of unit normal forces at s. These have been derived by the authors for several commonly used test specimen geometries. The singular integral equations (2) and (3) have been solved incrementally for the unknown cohesive stress $\sigma(s)$ and ℓ_p for given applied stress σ or K_I and materials properties G_F and $\sigma(w)$ until the ultimate stress σ_u at failure has been reached. σ_u has been found to vary with the size of the structure W and the crack size a ($\alpha = a/W$) as follows

$$\frac{\sigma_u}{f_t} = D_1(\alpha)\left(1+\frac{W/l_{ch}}{D_2(\alpha)}\right)^{-\frac{1}{2}} + \frac{D_1(\alpha)}{2D_2(\alpha)}\frac{W}{l_{ch}}\left(1+\frac{W/l_{ch}}{D_4(\alpha)}\right)^{-1} \tag{4}$$

where f_t and $l_{ch} = EG_F/f_t^2$ are the material properties which must be independently determined. The coefficients $D_1(\alpha)$, $D_2(\alpha)$ and $D_4(\alpha)$ are obtained by nonlinear regression of the test results on geometrically similar specimens of varying sizes. The formula (4) is found to be in excellent agreement with test data and to cover a very large size range of 1:80. In complete agreement with experimental observations, it predicts a weakening of the size effect as α reduces and a strengthening when W increases at constant α. The strength size effect is however never stronger than that predicted by the Griffith theory.