SIMULATING THE ORSZAG-TANG VORTEX USING RSPH

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Summary The formation of the compressible Orszag-Tang vortex has become a standard test for multi-dimensional MHD codes. In the present work, the problem is studied using the Lagrangian method Regularized Smoothed Particle Hydrodynamics (RSPH). First, a brief introduction to RSPH is given. Then, test results are presented, both for the case of uniform and non-uniform resolution. The results indicate that RSPH is a fully competitive, Lagrangian alternative to traditional grid-based methods for this type of problem.

INTRODUCTION

Regularized Smoothed Particle Hydrodynamics (RSPH) is an extension to the Lagrangian method Smoothed Particle Hydrodynamics (SPH) [1]. It was developed with the aim of achieving high accuracy modelling of hydrodynamic and magnetohydrodynamic problems using a highly adaptive numerical description [2]. In SPH based methods, a continuous fluid is represented by a discrete set of interacting particles moving with the fluid flow. Based on interpolation theory, the equations of motion can be expressed using integral interpolants, which in the discrete case are approximated by summation interpolants. The key parameter in determining the resolution of the interpolation is referred to as the smoothing length, denoted h [3].

RSPH differs from standard SPH in two important aspects. First, the former method allows h to vary both in time and space by orders of magnitude based on flexible, problem-specific criteria. The h-profile is chosen to be piecewise constant, with each step in space representing a factor of 2 change in h. Secondly, RSPH allows the particle distribution to be redefined at temporal intervals through a mass, momentum, magnetic energy, and total energy conserving process. Highly irregular particle distributions and particle penetration can thus be avoided, leading to reduced numerical errors. In comparison, standard SPH codes typically apply a uniform h-profile, or at best, allow h to vary slowly in time and space as a function of the particle number density. In addition, standard SPH does not provide any mechanism for optimizing the particle distribution during a simulation.

RSPH equations of motion

The ideal MHD model equations are found by neglecting the displacement current as well as the effects of electrical resistivity, viscosity and thermal conduction. The continuous equations are expressed in terms of the Lagrangian derivative $D/Dt = \partial/\partial t + \boldsymbol{v}\cdot\nabla$, and the units of the magnetic field are chosen so that the factor $1/\mu_0$ does not appear explicitly in the equations. For a given particle a, the discrete equations can be written as summations over neighbouring particles b:

$$\frac{d\boldsymbol{r}_a}{dt} = \boldsymbol{v}_a, \tag{1}$$

$$\rho_a = \sum_b m_b \mathcal{W}_{ab}, \tag{2}$$

$$\begin{aligned}\frac{d\boldsymbol{v}_a}{dt} &= \sum_b m_b \left(\frac{\boldsymbol{\mathcal{M}}_b}{\rho_b^2} + \frac{\boldsymbol{\mathcal{M}}_a}{\rho_a^2} + \Pi_{ab}\right)\cdot\nabla_a \mathcal{W}_{ab} \\ &\quad -\boldsymbol{B}_a \sum_b m_b \left(\frac{\boldsymbol{B}_b}{\rho_b^2} + \frac{\boldsymbol{B}_a}{\rho_a^2}\right)\cdot\nabla_a \mathcal{W}_{ab},\end{aligned} \tag{3}$$

$$\frac{de_a}{dt} = \sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{1}{2}\Pi_{ab}\right) \boldsymbol{v}_{ab}\cdot\nabla_a \mathcal{W}_{ab}, \tag{4}$$

$$\frac{d\boldsymbol{B}_a}{dt} = \frac{1}{\rho_a}\sum_b m_b (\boldsymbol{B}_a \boldsymbol{v}_{ab} - \boldsymbol{v}_{ab}\boldsymbol{B}_a)\cdot\nabla_a \mathcal{W}_{ab}, \tag{5}$$

$$P_a = (\gamma - 1) e_a \rho_a, \tag{6}$$

where m_a, $\boldsymbol{r}_a$, $\boldsymbol{v}_a$, ρ_a, P_a, $\boldsymbol{B}_a$, and e_a are the mass, position, velocity, mass density, thermal pressure, magnetic field strength, and internal energy of particle a. The term $\boldsymbol{\mathcal{M}}_a$ denotes the stress tensor given by

$$\boldsymbol{\mathcal{M}}_a = \boldsymbol{B}_a\boldsymbol{B}_a - \left(\frac{1}{2}\boldsymbol{B}_a{}^2 + P_a\right)\boldsymbol{\mathcal{I}}, \tag{7}$$

where $\boldsymbol{\mathcal{I}}$ is the unity tensor. The weight kernel used in this work, $\mathcal{W}_{ab} = \mathcal{W}(\boldsymbol{r}_a - \boldsymbol{r}_b, h_{ab})$, is the third order central B-spline with the characteristic scale length $h_{ab} = (h_a + h_b)/2$ [3]. The artificial viscosity term, Π_{ab}, is needed when modelling shocks [3]. The ratio of specific heat at constant volume and constant pressure is given by γ.

THE ORSZAG-TANG VORTEX

The 2D Orszag-Tang vortex problem was originally studied in the context of incompressible MHD turbulence [4]. Since then, a supersonic version of the problem has become a frequently used test of compressible MHD codes, see e.g. [5]. The test is set up in a 2D quadratic box with sides 1 length unit and periodic boundary conditions. The initial profiles of velocity and magnetic field are

$$\boldsymbol{v}(x,y) = [-\sin(2\pi y), \sin(2\pi x)] \tag{8}$$

and

$$\boldsymbol{B}(x,y) = \frac{1}{(4\pi)^{1/2}}[-\sin(2\pi y), \sin(4\pi x)], \tag{9}$$

respectively. The mass density and thermal pressure are initially uniform and equal to $25/(36\pi)$ and $5/(12\pi)$, respectively, and $\gamma = 5/3$.

In figure 1 the mass density is plotted at $t = 0.5$ for 4 different simulations. In the first 3 simulations, a uniform h-profile is used with an initial, rectangular lattice particle configuration of 128×128 (case a), 256×256 (case b), and 512×512 (case c) particles. One can see that the resolution is improved by increasing the number of particles and that the results fit well with earlier results [5]. In case d, a variable h-profile with a time-averaged particle number of roughly 47300, is used. Note that the resolution obtained in the central region in case d is comparable to that obtained in the same region in case c, despite the fact that more than 5 times as many particles have been used in case c.

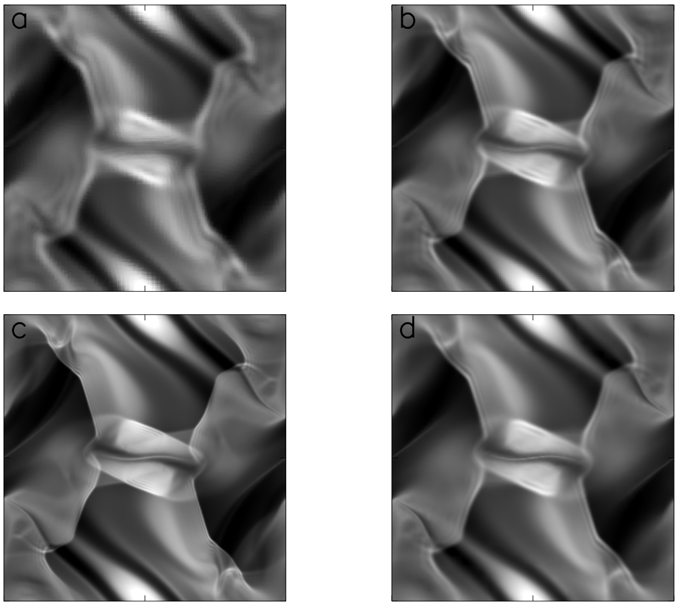

Figure 1. Mass density at $t = 0.5$ obtained from three different simulations using a uniform resolution of 128×128 (a), 256×256 (b), and 512×512 (c) particles, and a nonuniform resolution with a time-averaged particle number of roughly 47300 (d).

CONCLUSIONS

In this paper, the particle method RSPH has been used to study the Orszag-Tang vortex problem. Using a uniform h-profile, the accuracy of the presented results are comparable to that obtained with other methods. It is also shown that using a non-uniform h-profile, the time-averaged particle number can be reduced considerably with little reduction in accuracy. In problems where dominating features are more isolated than what is the case in the current problem, the benefit from using a non-uniform h-profile is expected to be even greater. Based on these results, RSPH appears to be a fully competitive, Lagrangian alternative to traditional grid-based methods when studying a large class of MHD problems.

References

[1] Gingold R.A., Monaghan J.J.: Smoothed particle hydrodynamics: theory and applications to non-spherical stars. *Mon. Not. R. Astron. Soc.* **181**:375–389, 1977.
[2] Børve S., Omang M., Trulsen J.: Regularized smoothed particle hydrodynamics: A new approach to simulating magnetohydrodynamic shocks. *Astrophys. J.* **561**:81–93, 2001.
[3] Monaghan J.J.: Smoothed particle hydrodynamics. *Annu. Rev. Astron. Astrophys.* **30**:543–574, 1992.
[4] Orszag S.A., Tang C.-M.: Small-scale structure of two-dimensional magnetohydrodynamic turbulence. *J. Fluid Mech.* **90**:129–143, 1979.
[5] Dai W., Woodward P.R.: On the divergence-free condition and conservation laws in numerical simulations for supersonic magnetohydrodynamic flows. *Astrophys. J.* **494**:317–335, 1998.