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HIDDEN AND DRIVEN SOLITONS IN MICROSTRUCTURED MEDIA

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Summary Wave propagation in microstructured materials is studied by making use of the Korteweg–de Vries type model equations. Model equation are solved numerically under harmonic initial and periodic boundary conditions. Existence of hidden solitons in conservative cases and emergence of driven solitons in nonconservative cases are demonstrated.

INTRODUCTION

Over the recent decade, the attention to microstructured materials has been arisen enormously. This concerns granular materials, polycrystalline solids, ceramic composites, functionally graded materials, alloys, etc. These advanced materials have found wide application in mechanical, electrical and computational engineering. In the present paper Korteweg–de Vries (KdV) type nonlinear evolution equations

$$u_t + [P(u)]_x + \delta_3 u_{3x} + \delta_5 u_{5x} = \alpha F(u), \quad P(u) = \gamma_2 u^2 + \gamma_4 u^4 \tag{1}$$

are used for modelling the 1D longitudinal wave propagation in nonlinear microstructured media. Here γ_2 is the second- and γ_4 the fourth-order nonlinear parameter while δ_3 is the third- and δ_5 the fifth-order dispersion parameter, x and t are space and time coordinates, respectively. The function $F(u)$ represents the influence of amplitude dependent external body force field. The most important point in this context is the possibility of emerging of solitons, i.e., solitary waves propagating without changing their shape and speed due to the balance of all physical effects (here between dispersion and nonlinearity). This important physical phenomenon was at first found in shallow water waves but nowadays is also found in solids, circuits, plasma, optical fibres, etc.
Our cases of interest are $\gamma_2 \neq 0$, $\gamma_4 \neq 0$, $\alpha = 0$ and $\gamma_2 \neq 0$, $\gamma_4 = 0$, $\alpha \neq 0$. The first case corresponds to alloy-type microstructured materials [1, 2], the second case — to a microstructured layer with energy influx like seismic waves in lithosphere [3]. With special scaling the examples studied below are $\gamma_2 = -0.5$, $\gamma_4 = 0.25$, $\alpha = 0$ and $\gamma_2 = -0.5$, $\gamma_4 = 0$, $\alpha \neq 0$, respectively. Both cases are analysed in order to cast light over the mechanism of possible emerging the solitary waves which bear the constant energy. The basic feature in both models is the existence of hidden solitons which can be amplified in the second case. Although our studies are confined to dynamics of solids, such mechanisms are of wider importance in solitonics.

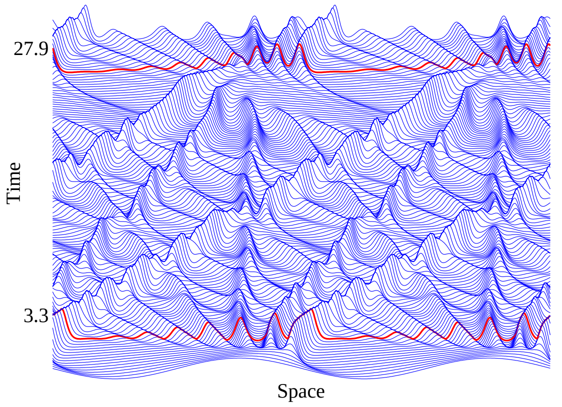

Figure 1. Interaction of KdV solitons: time slice plot over two 2π space periods ($d = 10^{-2.2}$).

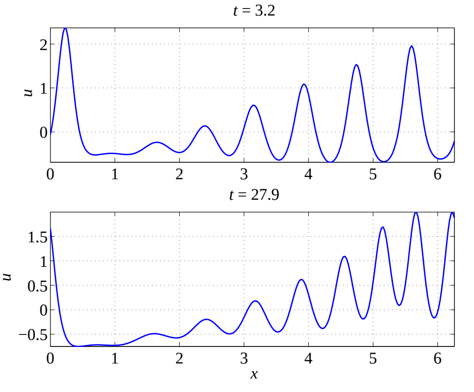

Figure 2. Initial train of 8 solitons at $t = 3.2$ and train of 9 solitons at $t = 27.9$.

HIDDEN SOLITONS

It is well known, that in systems governed by the KdV-type evolution equations the interaction of solitons causes phase shifts in soliton trajectories and simultaneous specific changes in their amplitudes (as a rule the amplitude of the higher soliton decreases during the interaction). If only single solitons are propagating in the media then the number of solitons is known and one can predict the location of a certain soliton at a certain time moment. However, in the case of arbitrary initial and periodic boundary conditions (harmonic, for example) a train of interacting solitons emerges. In this case the number of solitons taking part in the subsequent interaction process is usually higher than the number of solitons that can be determined making use of the initial train of equally spaced solitons. The solitons detectable from the initial train are called *visible*. In addition to these, some solitons can be hidden in the initial soliton train but they take part in the interaction process causing "additional" phase shifts and minima in trajectories and amplitude curves of visible solitons. Furthermore, when a certain number of interactions have taken place one can see in waveprofiles more solitons than initially during a short time interval. We call these small amplitude solitons *hidden*. Such a phenomenon can be found in the case of the KdV equation [4] as well as for our first case [5]. In Figs. 1 and 2 the existence of hidden solitons is demonstrated in the case of KdV equation. At time moment $t = 3.2$ the initial train of eight solitons (per one 2π period) is formed. However, at $t = 27.9$ one can observe a train of nine solitons.

In order to understand the essence of physical processes the actual number of solitons (including hidden ones) is of importance. The concept of hidden solitons is: (i) hidden solitons can emerge from arbitrary periodic initial excitation and they have small energy and amplitude; (ii) they cause changes, specific for soliton type interaction, in amplitudes and velocities of other solitons interacting with them; (iii) they can be detected in wave profiles for a short time interval only when several soliton interactions have been taken place and the equilibrium state is fluctuated, if ever; (iv) the physical essence of visible and hidden soliton is the same.

DRIVEN SOLITONS

In [6] the force field is given in by the cubic polynomial, i.e. $F(u) = \alpha u(u - \beta_1 u)(u - \beta_2)$, and the forced KdV equation is solved numerically under harmonic initial conditions. If $\alpha < 1$, then the influence of the field is weak which suppers some solitons in the KdV soliton train (including the influence of hidden ones). However for higher values of α the force field results in stationary solutions in the form of solitary- or cnoidal waves and no soliton interactions can be detected.

In the present paper the influence of periodic driving field $F(u) = \alpha \sin \beta u$ is discussed. Compared with the cubic field the influence of the periodic one is more complex. Now due to the driven field the KdV solitons can be suppressed or amplified to three different amplitude-levels. Solitons, amplified to different levels, have different velocities and therefore they interact (Figs. 3 and 4). The elastic character of these interactions shows their solitonic behaviour. In Fig. 3 the total number of amplified solitons (per one 2π period) is five: two solitons (amplified to the first level) are propagating to the left, two second level solitons and one third level soliton two the right. Other KdV solitons (including hidden ones) are suppressed (cf. Fig. 1). In Fig. 4 the total number of solitons is twelve: one first level soliton is going to the left and a group of eleven second level solitons to the right. Here the strength of the driven field is so high, that it introduces additional nonlinearity and dispersion to the system. Therefore the total number of amplified solitons is higher then in the corresponding KdV (conservative) case.

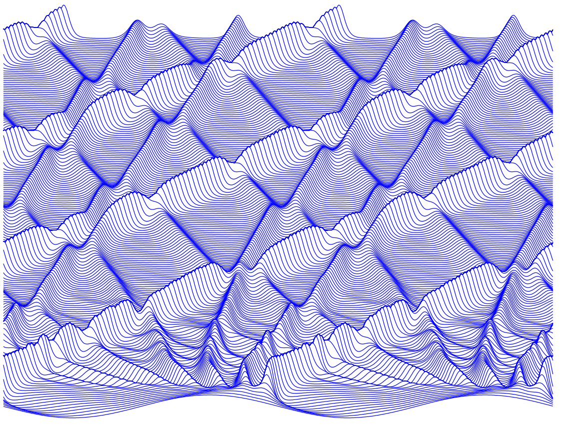

Figure 3. Emerging of driven solitons: time slice plot over two 2π space periods ($\alpha = 0.1$, $\beta = 7$, $d = 10^{-2.2}$)

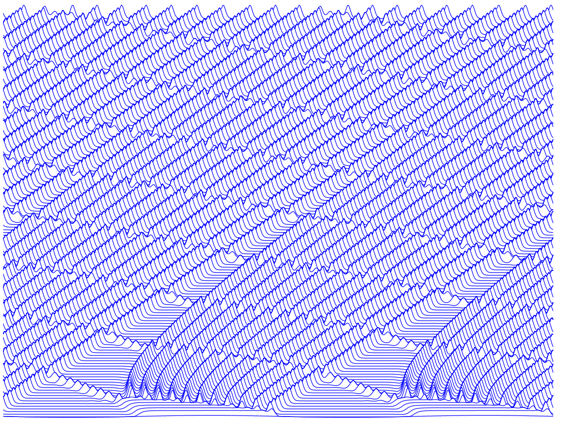

Figure 4. Emerging of driven solitons: time slice plot over two 2π space periods ($\alpha = 50$, $\beta = 1$, $d = 10^{-2.2}$)

CONCLUSIONS

In the conservative case the hidden solitons play a role in causing additional phase shifts in interaction processes, but do not play any significant role in energy sharing within the train of solitons. Their existence, however shows "possible energy pockets". In the nonconservative case those hidden solitons may be amplified under certain resonance conditions between driving field and the train of solitons. Such a phenomenon may open new possibilities for generating wave fields with designed properties. Energy conservation in such fields may be used in nondestructive testing of materials.

References

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