ENERGY SPECTRUM IN ROTATING TURBULENCE

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Summary
The transition between three-dimensional and quasi-two-dimensional turbulence in a rotating frame is experimentally investigated. Turbulence is generated by rapidly towing a grid in a rotating water tank, and the velocity field in a plane perpendicular to the rotation axis is measured by means of particle image velocimetry. During the decay, the power spectrum shows a power law $E(k) \sim k^{-n}$, with an exponent n that continuously increases from $n = 5/3$, as expected for the 3D Kolmogorov regime at large Rossby number, up to $n \simeq 2.2$ for the lowest Rossby number. This trend is consistent with the spectrum of the enstrophy cascade regime, $E(k) \sim k^{-3}$, in the limit of zero Rossby number.

INTRODUCTION

Turbulence subjected to system rotation, a situation present in a wide range of applications, from engineering to geophysics and astrophysics, is a complex problem which is not completely understood. The limit of very small Rossby numbers is expected to give rise to two-dimensional turbulence, where inverse energy cascade and enstrophy cascade may eventually take place. In the case of moderate rotation, the usual description in terms of direct energy cascade should be modified to account for the effect of the Coriolis force.

Adding rotation to an otherwise 3D homogeneous turbulence introduces a new characteristic time, Ω^{-1}, that may affect the largest scales associated with low levels of vorticity compared to the background vorticity 2Ω. On this phenomenological ground, Zhou (1995) [1] proposes to use the timescale Ω^{-1} instead of the characteristic turn-over time based on the Kolmogorov estimate, leading to a k^{-2} power law spectrum for the small wavenumbers instead of the classical $k^{-5/3}$ law. However, taking account of the anisotropy leads to a k^{-3} law, in accordance with the enstrophy cascade regime expected at large wavenumbers in the limit of zero Rossby number [2].

EXPERIMENTAL SET-UP

The experimental cell, similar to the one from Hopfinger *et al.* (1982) [3], is sketched in figure 1(a). It consists of a square water tank, 55 cm in height and 35 cm in side, mounted on a rotating turntable, whose angular velocity Ω can be adjusted between 0 and 4.5 rad/s. A cover is placed below the free surface to avoid surface waves and unwanted β-effects. A turbulent velocity field is generated by rapidly towing a grid through the height of the tank. The grid consists of 1 cm square bars with a mesh $M = 39$ mm. Typical velocity fluctuations $u' \simeq 0.1$ m/s are obtained by imposing a grid velocity of order of 1 m/s. During the turbulence decay, the background rotation gradually affects the entire flow in an homogeneous way.

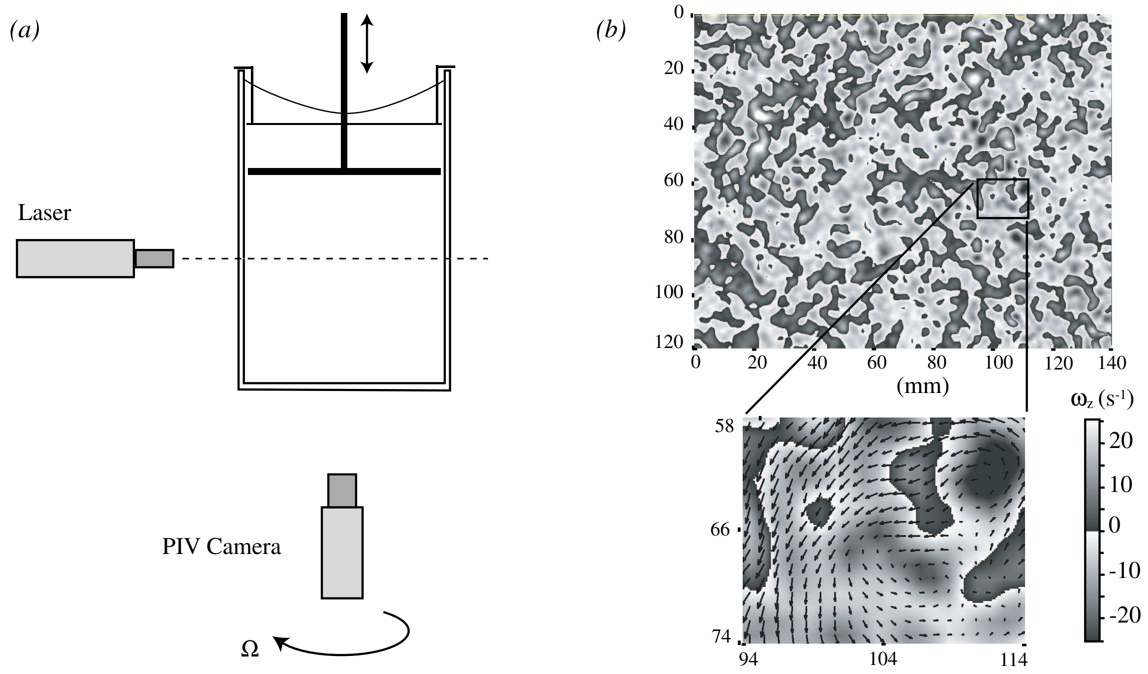

Figure 1. (a) Schematic of the experimental set-up. The water tank, the oscillating grid and the camera are in the rotating frame, while the pulsed laser remains in the laboratory frame. (b) Example of vertical vorticity field measured by PIV.

Instantaneous velocity fields in the horizontal plane (x, y) are obtained using a particle image velocimetry (PIV) system. The flow is seeded with small glass spheres, and illuminated by a horizontal laser sheet produced by a double pulsed laser. Images are acquired with a double-buffer high resolution camera in the rotating frame, synchronized with the laser at a maximum rate of 4 frames pair per second.

The instantaneous velocity fields may be characterized by the Reynolds number, $\mathrm{Re}_M = u'M/\nu$, and the *macro* Rossby number, $\mathrm{Ro}_M = u'/2\Omega M$, based on the velocity fluctuation u' and the mesh size M. As the turbulence decays, both numbers decrease, with the ratio $\mathrm{Ro}/\mathrm{Re} = \nu/2\Omega M^2$ remaining constant. In addition, a *micro* Rossby number, $\mathrm{Ro}_\omega = \omega'/2\Omega$ is introduced. We are interested in the regime $\mathrm{Ro}_M < 1$ and $\mathrm{Ro}_\omega > 1$, i.e. the intermediate regime where the large scales are strongly affected by the background rotation, whereas the small scales remain essentially three-dimensional.

RESULTS

Figure 2(a) shows three power spectra, measured in the horizontal plane, during the decay of the turbulence, for $\Omega = 4.5$ rad s^{-1} and a grid velocity of $V_g = 0.8$ m s^{-1}. For the first spectrum, the Rossby number is $\mathrm{Ro}_\omega = 1.1$, and a pure Kolmogorov scaling law can be seen, $k^{-5/3}$, indicating a weak influence of the rotation. As the Rossby number decreases in time ($\mathrm{Ro}_\omega = 0.53$ and 0.29), the spectra show a steeper slope, k^{-n} with $n > 5/3$.

The exponent n of the power spectrum is shown in figure 2(b) as a function of Ro_ω. The collapse of the curves for the different data sets, corresponding to various initial Reynolds numbers, is remarkable, indicating that the micro Rossby number is the relevant local parameter for the energy spectrum. The exponent is found to continuously increase for $\mathrm{Ro}_\omega < 1$. The value $n = 2$ is observed, but it only appears as a transient state in the route towards the bidimensionalization of the flow. The trend is found consistent with the $n = 3$ limit, expected for purely 2D turbulence in the enstrophy cascade regime.

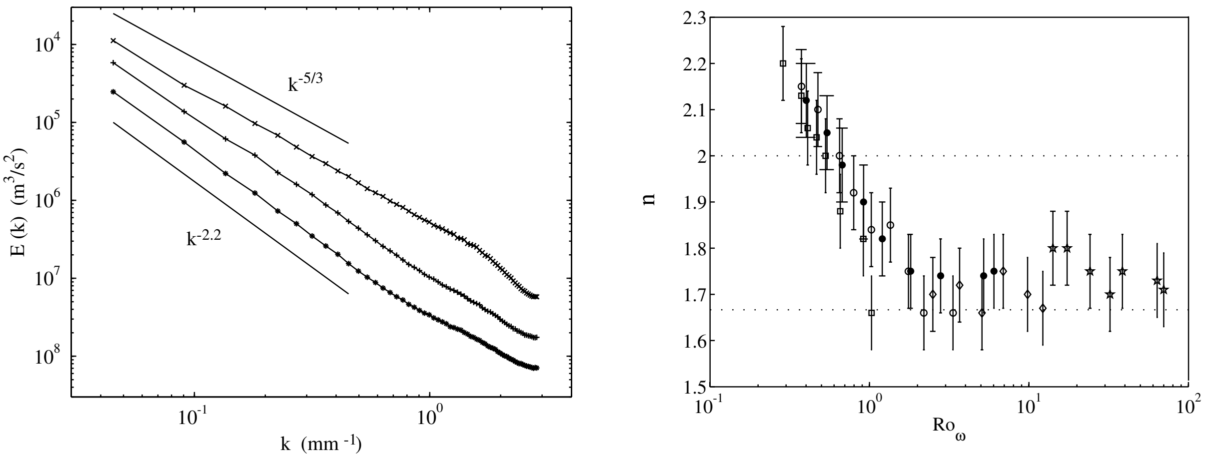

Figure 2. (a) Power spectra in the horizontal plane. (b) Exponent n of the power spectrum as a function of the micro Rossby number $Ro_\omega = \omega'/2\Omega$.

References

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[3] Hopfinger E. J., Browand F. K., Gagne Y.: Turbulence and waves in a rotating tank. *J. Fluid Mech.* **125**:505, 1982.

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