EFFECT OF A "STATIC" RESONANCE IN ELASTIC THIN-WALLED CYLINDERS

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Summary. Basing on the experiments the conclusion is drawn about existence of "static" resonance phenomena for shells at presence of periodic non-homogeneity of stress-strain state. The essence of resonance can be explained as a development, at static loading of thin-walled compressed cylinder of large bending deformations in case of variability of non-homogeneity, which is close to variability of first mode of eigen lateral oscillations of a structure.

The experiments data are considered in which deformation and buckling of longitudinally compressed circular cylindrical shells under periodically non-homogeneous in circular direction pre-buckling stress-strain state (SSS) are studied. The estimations have been performed on elastic, smooth, longitudinally reinforced (stringer type) thin-walled cylinders. The non-homogeneity of pre-buckling SSS of smooth shells was caused either by non-uniformity of loading or by non-uniformity of initial imperfections in the middle surface shape. For stringer modules the non-uniformity was caused by change of sign of eccentricity of ribs allocation. All experiments have been performed on high-quality small-sized samples using the same methods. These samples have been made produced from steel strip X18H9n (*conventional yield strength* s_{02} *= 800 MPa)* using similar technology of spot weld. Given loading scheme and boundary conditions of simple support have been provided with special edge testing devices.

1. Non-uniform compression of smooth shells. One learnt the double-sided (on both edges of a shell) piece-wise-uniform kinematic (the equal displacements were applied) longitudinal compression on segments along the guiding circle. The compression is uniformly distributed along the perimeter. The dimensions of the loaded and unloaded segments were taken equal, and their allocation on the edges was accepted symmetric with respect to the cross-section of the cylinder. The number of segments of the loading (p), corresponding to the variability of the load in circumference direction, varied in a wide range: p = 2-48. The samples with the following relative geometrical characteristics were tested: L/R = 0.3-3.5; R/h = 150 - 300 (L, h are the length and thickness of the shell).

At prebuckling stage under uniform axisymmetric compression of shells the bending deformations were observed, which were caused by Poisson's effect together with the initial deflection development. The loss of stability occurred as a continuous process with one, snap creating 2-4 belts of rhomb shaped dimples. At non-uniform compression, alongside with a non-homogeneity of membrane stresses field the essential periodic in a circumferential direction bending deformation were observed. The character and intensity of these deformations were determined by changing the loading p and the geometry of a shell. The prebuckling radial displacements (w) appeared to be periodic and high stable. For example, for shells with geometry L/R = 2.0; R/h = 300 for all types of prebuckling deformation (excluding the case p = 2) the variability of displacements in a circumference direction corresponds with variability of the loading p. In the loaded zones the displacements develop inside the shell and in the unloaded zones– outside the shell. At 5<p<18 these displacements can be well described by dependence $w=W\cos(py/R)$. The most intensive bending takes place at p = 6. The maximum curvature of generating line in this case is observed in a middle part of the shell. The increase of p leads to the movements of a zone with maximum curvature to the edges. At p> 24 the non-uniformity of the loading does not cause any peculiarities to the prebuckling behaviour of the shells. In case of small variability of the loading (at p< 5) the bend of generating line occurs along the whole length of a shell, however its intensity is much lower, than at p = 6. Note that at p = 2 the harmonics with a number of waves n= 6 was the basic components of the prebuckling bend in a circumference direction. The load-bearing capacity for non-uniformly compressed shells was reached because of the loss of stability in elastic stage of deformation. The values of N^{cr}, as well as prebuckling deformations, were determined varying the load p. The increase of a critical load (N^{cr}) at increase ð has appeared non-monotone, and the minimum value of N^{cr} in all range ð is marked at maximum prebuckling deformations (ð=6).

The peculiarities of the waveformation at buckling are determined by the type of prebuckling deformation**.** In case p = 6 the buckling mode was similar to mode of the first tone of lateral oscillations of a shell: the half-wave of dimples, spread along the whole length of the cylinder.

Similar results were obtained on the samples with other geometrical characteristics. The most considerable decrease of the critical stresses and loads at the most essential prebuckling deformations is observed. The deformations are characterized by the bending of generating lines on all length of the shell, and by the waveformation, similar mode of the first tone of lateral oscillations of a shell (n – the number of waves in circumference direction). The analysis of results, obtained on samples of the different geometry, showed, that they can be generalized if the variability of the loading in circumference direction connect with variability of the mode the first tone of oscillations n of a shell the shell. According to it the "resonance" variability of the loading, at which one the minimum critical stresses and loads is realized, can be evaluated as $p^*=n$. The generalized dependence "N^{cr}/N^{c}-p/n" (N^{c} is the classical value of critical load at homogeneous compression) is presented in fig. 1.

2. Smooth shells with imperfections. The experiments were performed on samples with the following geometry: L/R=2, R/h=360. The initial imperfections of a middle surface, in a form local flat dimples with depth W_0=(1.5-2.0) h, were made on shells artificially – by indenting of the spherical segment in the cylinder from its external surface (since the natural sag was negligible, it was suppressed by the artificial sag). These dimples were applied in the middle (by length) cross-section, uniformly along the circumference. The number of dimples (variability of periodic imperfections in a circumference direction -n^0) varied from 2 up to 17. The samples without artificial dimples lost their stability with one snap at average values of N^{cr}=0.61N^c. The variability of the postbuckling mode – n^{cr}=9-10 (the calculated number of waves corresponding to N^c is n^c =17). The local buckling preceded to the general loss of stability of shells with artificial imperfections in all cases. The relative loads of transfer to the postbuckling configurations, depending on n^0, are represented in fig. 2. The light circles correspond to the loads of local buckling, the light quadrates ones to bifurcation modification of the local postbuckling configuration, and the dark circles correspond to the general loss of stability.

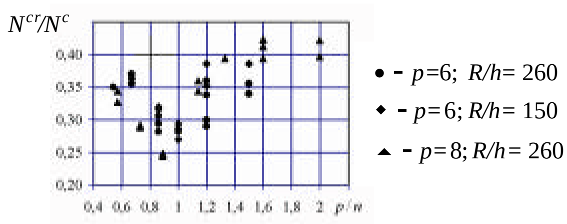

Fig.1

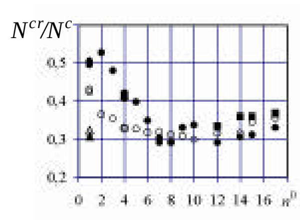

Fig.2

The analysis of dependence "$N^{cr}/N^{\tilde{n}} – n^0$" shows, that there is a selectivity of longitudinally compressed of the shell to the variability of the considered imperfections n^0. The highest influence of imperfections on N^{cr} appears at n^0=7-8. This variability of imperfections corresponds to the variability of the mode of the first tone of lateral oscillations of shell of buckling of the cylinder at external radial pressure, the designed value by which one compounds n = 7. In this case the buckling occurred with creation of the large dimples, similar of the mode of lateral oscillations. It should be noted that the presence of the essentially localized in a longitudinal direction of the initial dimple on a shell lead to the local postbuckling mode. The initial dimple was close to axisymmetric and differs by significant amplitude ($W_0 \geq 0.6h$). The light triangle corresponds to load of transfer to the local postbuckling mode, the dark triangle – to the general postbuckling mode.

3. Stringer shell with an eccentric versatile reinforcement. The series of 9 high-quality shells of equal geometry were tested. Each shell was equipped with 36 stringers, equidistant from each other: two samples with one-sided external and internal ribs and 7 samples - with versatile allocation of ribs periodically located in a circumference direction. The dimensions of panels with external and internal equipment were equal. The stiffeners allocation is shown in the table, where k_+, k_- are the numbers of panels on a sample with internal and external allocation of ribs respectively; t- number of ribs placed on each panel. It should be noted that the sign of an eccentricity of ribs in a circumference direction periodically varied for shells with a versatile reinforcement with frequency k=k_+. The geometry of shells: 2R=172 *mm*; (L/R=1); (R/h=465). The stringers of ⌊-profile with dimensions of cross-sections (3.2×3.9×0.345) *mm* were connected to the skin along the wide side of angle profile.

The local buckling of the skin between the ribs foreran to the overall loss of stability of shells. Such local buckling occurred with creation of large dimples with one half-wave along the length. The levels of the loads for which the intensive local waveformation (N^m) started, are presented in the Table 1.

Table 1

1 of sample	1	2	3	4	5	6	7	8	9
k_+/k_-	1/0	0/1	18/18	9/9	9/9	6/6	4/5	3/3	2/2
t	36	36	1	2	2	3	4	6	9
N^m, kN	36.5	32.3	22.3	12.1	13.3	22.0	21.9	22.4	22.2
N^{cr}, kN	41.4	59.1	31.9	24.6	26.2	40.8	30.1	46.7	31.9
N^*, kN	45.9	59.1	51.7	47.7	48.9	48.1	52.2	54.5	54.4

The collapse of the shells occurred during the process of their sequential loading. At first, the dimples were created "softly" without reduction of the load in zones with internal ribs. The corresponding load in the table is marked as N^{cr}. The limit of local-bearing capacity was reached in all cases with a destruction of external ribs. Such destruction was followed by sharp increase of intensity of buckling due to appearance of zones with plastic deformations. It should be noted that the ultimate loads (N^*) are insignificantly dependent on k. At the same time the value of N^{cr} sharply responds to change k and achieves of minimum value (twice less of N^*) at the most intensive buckling. The value of k is close to variability of general mode of the first tone of lateral oscillations of a shell (n = 11).

Conclusion. The reduced three examples indicate onto essential increase of static lateral deformations of a shell, at the variability of non-homogeneity close to the variability to the first mode of eigen oscillations. Such effect can be called a “static” resonance. It is possible to suppose, that the effect of a static resonance is inherent, to some extent, to all two-dimensional thin-walled structures. It is obvious, that the presence of such negative effect, as a static resonance, should be taken into account at the design of structures.