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ON THE STABILITY OF THE SKY-HOOK

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Summary The sky-hook, that is a string forming a connection from the surface of the Earth to a satellite in geostationary orbit, which may be used as track for an Earth to space elevator, is an old dream of mankind, originating about 100 years ago in Russia. Besides the question of feasibility from a technological point of view also the question concerning the stability of such a configuration has not yet been completely solved. Under the assumption that a proper material (carbon nanotubes) is available, making the connection possible from the technological point of view, we address the question of stability of the radial relative equilibrium of a very long tapered string, which rotates synchronously with the Earth and reaches from the surface of the Earth up into the sky. The solution of the stability problem for different string materials is given by application of the Reduced Energy Momentum Method. Since the string is orbitally unstable, we connect it to a satellite in geostationary height and give the minimum mass of the satellite to stabilize the whole configuration.

MOTIVATION AND HISTORY OF THE PROBLEM

One of the main problems of modern space exploration and space technology is the high cost of sending a payload from the surface of the Earth into space. Depending on the destination in space these costs were about $10^4 - 10^6$ US Dollars for one kilogram of payload in the year 2000.

Hence for a long time there have been other ideas around for a cheaper way of transporting payloads into space orbit. In 1960 the Russian scientist Yu. Artsutanov ([1]) proposed the very interesting idea to build a celestial elevator from the surface of the Earth to a satellite in geostationary orbit. In order to compensate for the weight of the string, the string must be extended beyond the geostationary radius (35863 km altitude above the Earth). The part of the string extending beyond the geostationary orbit must have a length, which is several times the length to the geostationary orbit ([3]) if it is designed for minimum weight (see also Fig. 2).

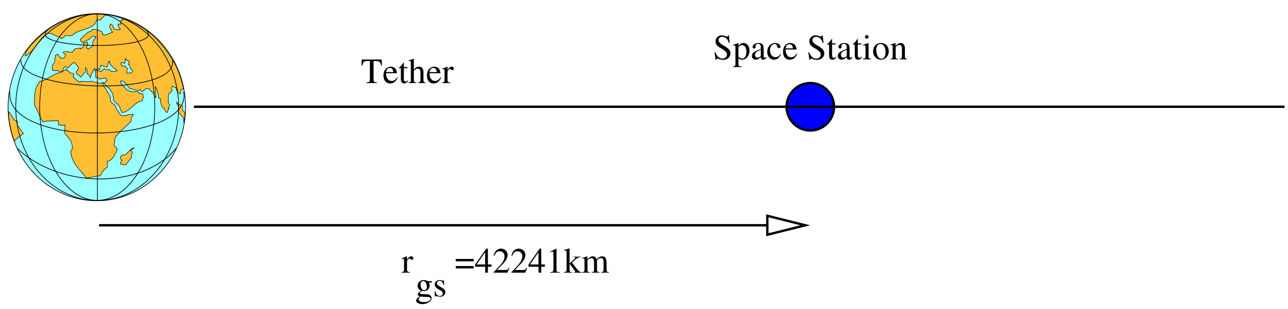

Figure 1. String connecting a satellite in geostationary orbit with the surface of the Earth. As counterweight another string must be deployed outside the geostationary height. For a minimum weight design its length and its shape are given in Fig. 2.

Artsutanov's idea makes use of the fact that a massive string moving on a circular orbit around the Earth, under the action of gravitational and centrifugal forces, finally will reach a relative equilibrium position, which is its radial position. It can be shown that in this equilibrium position the string is under tension ([2]). In order to obtain a minimum weight design the shape of the string must be tapered such that it is thickest at the point of highest tension which is at the geosynchronous radius and thinnest where the tension is lowest (at its ends) (Fig. 2). Such a string configuration could be used as track for a space elevator to provide easy transportation of payloads to the geosynchronous orbit and beyond ([6]).

REALISATION AND STABILITY ANALYSIS

Until 1991 these ideas were purely academic, since no material was available to realize such a project. However, at that time so-called "carbon nanotubes" ([7]) were discovered, which are cylindrical macromolecules composed of carbon atoms, which are formed from a flat periodical hexagonal latice with the thickness of the size of an atom. Single walled nanotubes have been produced with a diameter of a few nanometers ($1[nm] = 10^{-9}[m]$) and a length of the order of Centimeters. Hence sofar an aspect ratio of order 10^7 has been reached. Single walled nanotubes form the building block of multi-walled nanotubes. Moreover, it is conceived to have bundels of nanotubes. Forming nanoropes from nanotubes a theoretical strength of 100 times higher than steel can be expected but with only one-sixth of the weight of steel. Moreover, besides their extreme strength they also allow large strains, up to 16-24 % ([7]).

The ratio between tensile strength and density is crucial for the taper ratio of the string, that is, the ratio between the cross sectional area of the string at the geosynchronous orbit to the cross sectional area at Earth. For example from the calculations performed in [3] and our calculations (Fig. 2) the taper ratio required for steel would be 1.7×10^{33}, for Kevlar 2.6×10^{8} and for carbon nanotubes (theoretical strength) only 1.5 .

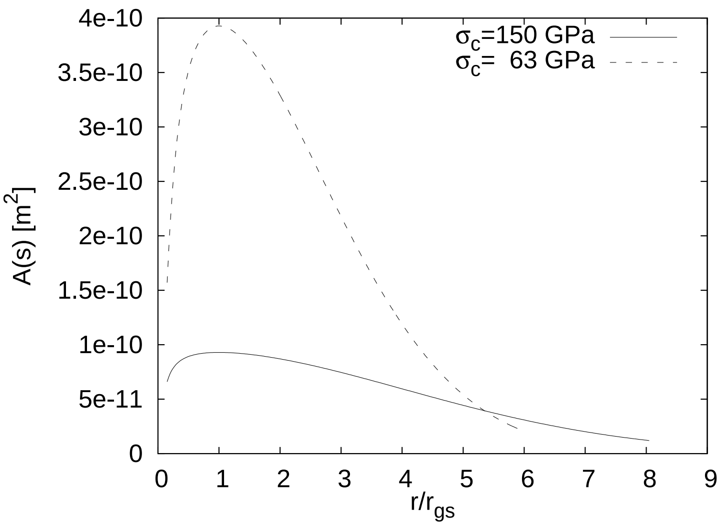

Figure 2. For two strings in the radial relative equilibrium configuration on the geostationary orbit around the Earth the areas of their cross-sections for carbon nanotubes with different admissible tensile stresses σ_c are given. The curve denoted by $\sigma_c = 150$ GPa corresponds to nanotubes of theoretical maximum strength resulting in a strain of $\epsilon = 0.238$, whereas the curve denoted by $\sigma_c = 63$ GPa corresponds to nanotubes of the strength measured experimentally in [7]. The corresponding strain is $\epsilon = 0.1$. For both strings the calculations habe been performed with endmasses of 1 kg. Moreover the ratio between the cross-sectional area at the geostationary hight to the area at the surface of the Earth follows from the figure to 1.41 for $\sigma_c = 150$ GPa and 2.50 for $\sigma_c = 63$ GPa.

A careful, practically relevant investigation of the stability of the system's relative equilibrium would have to take into account various perturbations such as the gravitational attraction of the moon, atmospheric drag and payloads moving up and down the string ([3], [6]). However, even the simpler question of stability of a long unperturbed tapered string in the spherical symmetric Newtonian gravitational field is still an open problem, because in [2] and [4] it is shown that a dumbell satellite, which is a system of two point masses connected by a massless rigid rod, possesses a stable radial relative equilibrium position on a circular orbit around the Earth only if the distance between the two masses is not too large. Here by large we understand a distance of the order of the radius of the orbit.

Hence the purpose of this paper is to investigate whether a continuous massive string, necessary for the realization of the skyhook, has a stable radial relative equilibrium. Since it will turn out that the radial relative equilibrium position of the tapered string is unstable we include at the geostationary height a satellite and ask the question, whether it has a stabilizing effect. Since this is the case we give the answer to the next question what is its minimum necessary mass. The proper theory to answer these questions is the Reduced Energy Momentum Method ([4], [8], [5], [10]). For details see [9].

CONCLUSIONS

Our main result is that the radial relative equilibrium position of a massive tapered string moving in geostationary orbit around the Earth, made of the new material called "carbon nano tubes", reaching from the surface of the Earth beyond the geostationary radius, is orbitally unstable. However, if the string is connected to a massive satellite moving in geostationary height the whole system can be stabilized. The minimum value of the mass of the satellite to achieve this stabilizing effect for the radial configuration is given in [9].

ACKNOWLEDGEMENT

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